APMA 2811D- Asymptotic Problems for Differential Equations and Stochastic Processes

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Course web-page: http://www.dam.brown.edu/people/kspiliop/F09APMA2811D.html
Meets: TBA
Office hours: TBA
Text: I will mostly use the book of G. A. Pavliotis and A. M. Stuart on Multiscale Methods: Averaging and Homogenization, Springer, 2007. Moreover, notes will be provided for most, if not for all, of the lectures.
Recommended textbooks:

- For multiscale methods and perturbation theory:

  1. G. A. Pavliotis and A. M. Stuart, Multiscale Methods: Averaging and Homogenization, Springer, 2007
  2. A. Bensoussan, J. -L. Lions and G. Papanicolaou, Asymptotic Analysis for Periodic Structures (Studies in mathematics and its applications), Elsevier, 1978
  3. H. Cheng, Advanced Analytic Methods in Applied Mathematics, Science, and Engineering, Luban Press, 2005.
  4. M. H. Holmes, Introduction to Perturbation Methods, Springer, 1998.

- For stochastic calculus and the interplay between PDE's and stochastic processes:

  1. M. Freidlin, Functional Integration and Partial Differential Equations, Princeton University Press, 1985
  2. B. Oksendal, Stochastic Differential Equations: An Introduction with Applications, Springer, 2007 (6th edition)
  3. I. Karatzas and S. E. Shreve, Brownian Motion and Stochastic Calculus, Springer, 2nd edition

In regards to the WKB method and boundary layer theory, I will also make use (in addition to the recommended textbooks mentioned above) of a collection of notes that were generously provided to me by Professor Dionisios Margetis at University of Maryland at College Park.
Course Description: Concepts and analytic & probabilistic tools used in various scientific disciplines. Emphasis will be placed on

1. Asymptotic problems for ODE's and PDE's: approximate solutions of problems that have small (or large) parameters or variables (WKB method, boundary layer theory)

2. Review of probability theory, introduction to stochastic calculus (Brownian motion, stochastic differential equations, Itô formula, Fokker-Planck eqs, Feynman-Kac formula, relation to PDE's)

3. Homogenization for PDE's and stochastic processes using various deterministic and probabilistic tools. Homogenization for diffusion in periodic and random media. Averaging for SDE's. Multiple scales method and two-scale convergence.

The course material will be based on theory, methods and examples from various scientific disciplines.

Objective: To learn various analytic and probabilistic techniques which are useful in the analysis of ODE's, PDE's and SDE's that depend on small (or large) parameters and have rapidly oscillating coefficients. To apply techniques from perturbation theory to study homogenization problems for PDE's and SDE's. To derive rigorous proofs of the formal calculations.

Grading: No exams and no tests. The grade will be based on a few practice problems. Each problem set will be due (usually) 2-3 weeks after the date it is handed out.