GRS MA 777- Multiscale Methods for Stochastic Processes and Differential Equations

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Meets: Tuesday-Thursday 12:30-1:45 at CAS 316
Office hours: 11-12 on Tuesday-Thursday

Textbooks:

- Required textbook: G. A. Pavliotis and A. M. Stuart, Multiscale Methods: Averaging and Homogenization, Springer, 2007
- Recommended textbook: I. Karatzas and S. E. Shreve, Brownian Motion and Stochastic Calculus, Springer, 2nd edition, 1991

Course Description: Data obtained from physical systems very often possess many characteristic length- and time-scales. In such cases, it is desirable to construct models that are effective for large-scale structures, while capturing small scales at the same time. Modeling this type of data and physical phenomena via multiple scale diffusion processes and differential equations with multiple scales may be well-suited in many cases. Thus, such models have been used to describe the behavior of phenomena in scientific areas such as chemistry and biology, ocean-atmosphere sciences, finance and econometrics.

In this course, we will study concepts, analytic and probabilistic tools that are used in various scientific disciplines. Emphasis will be placed on

1. Review of probability theory, introduction to stochastic calculus (Brownian motion, stochastic differential equations, Itô formula, Fokker-Planck eqs, Feynman-Kac formula, relation to PDE's)
2. Multiple scale methods (averaging and homogenization) for stochastic processes and PDE's using various deterministic and probabilistic tools.
3. Numerical methods, Monte Carlo methods and statistical inference methods for multiscale processes.
4. Applications to various disciplines such as mathematical finance, physics, chemistry and engineering will be discussed.

The course material will be based on theory, methods (both theoretical and computational) and examples from various scientific disciplines.

Course Prerequisites: Introduction to probability and stochastic processes (MA 581 and MA583 or equivalents) and Differential Equations (MA226 or MA231 or equivalent). PDE's, graduate level probability and statistics will be helpful but NOT necessary. Students are expected to have the knowledge equivalent to undergraduate level probability, stochastic processes and differential equations.

Tentative Course Syllabus:

- Week 1: Introduction & Overview

  Why multiscale models? Examples of physical systems that exhibit many different length and time scales. Overview of course material and course requirements.

- Week 2: Warm-up methods: asymptotic problems for ODE's.

  Approximate solutions of problems that have small (or large) parameters or variables (WKB method)

- Week 3: Asymptotic problems for ODE's.

  Introduction to singular perturbation methods and boundary layer theory for simple ODE models.

- Week 4: Review of probability theory and stochastic processes

  Review of basic notions in probability theory, such as expected value, variance, conditional expectation, stochastic processes, Markov property, Brownian motion, martingales and basic inequalities and different modes of convergence.

- Week 5: Quick introduction to stochastic calculus.

  Stochastic integral, Ito stochastic process, stochastic differential equations, Ito formula, exponential martingales, Girsanov's theorem for change of measure, connections to PDEs, Fokker-Planck equation, Feynman-Kac formula, invariant measure and ergodicity.

- Week 6: Averaging principle

  Motivation from WKB method. Applying averaging principle to slow-fast systems of stochastic differential equations, coarse-graining and dimension reduction, rigorous proofs and examples.

- Week 7: Homogenization for stochastic processes and PDEs

  Homogenization theory for complex multiscale stochastic differential equations, coarse-graining, derivation via multiscale expansion of the backward Kolmogorov equation, cell problem, rigorous proofs and examples.

- Week 8: NO class, BU spring break.

- Week 9: Homogenization (cont.)

  Homogenization for elliptic and parabolic Partial differential equations using probabilistic methods and Feynman-Kac formula, rigorous proofs and examples.

- Week 10: Homogenization (cont.)

  Homogenization for elliptic and parabolic partial differential equations using analytic methods, Lax-Milgram theorem, Fredholm alternative and two-scale method, rigorous proofs and examples.

- Week 11: Numerical methods:

  Numerical methods for efficient simulation of multiscale systems, resolving the different scales, Monte-Carlo methods and statistical calibration methods.

- Week 12: Applications

  Applications from various scientific disciplines such as: diffusion in a rough potential and molecular dynamics, non-linear oscillators and bifurcation diagrams, climate models, solid-state physics and ferromagnetism.

- Week 13: Applications (cont.)

  Multiscale methods in mathematical finance, volatility time scales and option-pricing under different time scales.

- Week 14: Presentation of class projects

- Week 15: Presentation of class projects

Assigned work and grading: Registered students will be expected to do regular reading, complete a few sets of homework problems (approximately biweekly) (75%) and present a final project (25%). Needless to say, you should work on the homework on your own, unless otherwise instructed by me. Late homeworks will NOT be accepted. The grading policy may change depending on the progress of the class.

Please Note: Students are responsible for knowing the Boston University Academic Conduct Code which is posted at

`http://www.bu.edu/academics/policies/academic-conduct-code/`