CAS MA884- Topics in Multiscale Analysis: Theory, Computation and Applications

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Course web-page: http://www.math.bu.edu/people/kspiliop/Spring2013MA884.html
Meets: Spring 2014, Tuesday-Thursday 9:30-11:00
Office hours: after class
Text: I will mostly use the book of G. A. Pavliotis and A. M. Stuart on Multiscale Methods: Averaging and Homogenization, Springer, 2007. Moreover, notes will be provided for most, if not for all, of the lectures.
Recommended textbooks:

- For multiscale methods and perturbation theory:

    1. G. A. Pavliotis and A. M. Stuart, Multiscale Methods: Averaging and Homogenization, Springer, 2007
    2. J.-P. Fouque , G. Papanicolaou, R. Sircar, K. Solna, Multiscale Stochastic Volatility for Equity, Interest Rate, and Credit Derivatives, Cambridge University Press, 2011
    3. A. Bensoussan, J. -L. Lions and G. Papanicolaou, Asymptotic Analysis for Periodic Structures (Studies in mathematics and its applications), Elsevier, 1978
    4. V.V. Jikov, S.M. Kozlov, O.A. Oleinik, Homogenization of Differential Operators and Integral Functions, Springer-Verlang, 1991
    5. H. Cheng, Advanced Analytic Methods in Applied Mathematics, Science, and Engineering, Luban Press, 2005.
    6. M. H. Holmes, Introduction to Perturbation Methods, Springer, 1998.

- For stochastic calculus and the interplay between PDE's and stochastic processes:

    1. M. Freidlin, Functional Integration and Partial Differential Equations, Princeton University Press, 1985
    2. B. Oksendal, Stochastic Differential Equations: An Introduction with Applications, Springer, 2007 (6th edition)
    3. I. Karatzas and S. E. Shreve, Brownian Motion and Stochastic Calculus, Springer, 2nd edition

Course Description: Data obtained from a physical system sometimes possess many characteristic length and time scales. In such cases, it is desirable to construct models that are effective for large-scale structures, while capturing small scales at the same time. Modeling this type of data and physical phenomena via mulitple scale diffusion processes and differential equations with multiple scales may be well-suited in many cases. Thus, such models have been used to describe the behavior of phenomena in scientific areas such as chemistry and biology, ocean-atmosphere sciences, finance and econometrics.

In this course, we will study concepts, analytic and probabilistic tools that are used in various scientific disciplines. Emphasis will be placed on

1. Review of probability theory, introduction to stochastic calculus (Brownian motion, stochastic differential equations, Itô formula, Fokker-Planck eqs, Feynman-Kac formula, relation to PDE's)
2. Multiple scale methods (averaging and homogenization) for stochastic processes and PDE's using various deterministic and probabilistic tools.
3. Backward SDE's and their application to homogenization of related PDE's.
4. Numerical methods and Monte Carlo methods for multiscale processes.
5. Applications to various disciplines such as mathematical finance, physics, chemistry and engineering will be discussed.

The course material will be based on theory, methods (both theoretical and computational) and examples from various scientific disciplines.

Course Prerequisites: The course will be largly self-contained, accesible to a broad audience and a revision to the basic tools from probability, stochastic processes and differential equations that are needed, will be given. However, students are expected to have the knowlesge equivalent to undergrdauate level probability, stochastic processes and differential equations. PDE's and graduate level probability will be helpful but NOT necessary.

Grading: No exams and no tests. The grade will be based on a presentation and on a few practice problems. Each problem set will be due (usually) 2-3 weeks after the date it is handed out.