MA225C Mock Final Exam

Name: ________________________________ (1 mark)

TRUE/FALSE. Write 'T' if the statement is true and 'F' if the statement is false. (11 marks)

1) Let f(x,y,z) be a function. The line integral of curl(∇ f)) around any circle is zero. 1) ________

2) If the line integral of a vector field along the closed loop $x^2 + y^2 = 1$, $z = 0$ is zero then the vector field is conservative. 2) ________

3) The flux of the curl of a vector field through the disc $x^2 + y^2 \leq. 1$, $z = 0$ is always zero. 3) ________

4) If a function f(x,y) has a critical point at (0,0) then div(grad(f))(0,0) is zero. 4) ________

5) The flux of a vector field F with length $|F| = 1$ through a triangular surface S can not be larger than the surface area of S. 5) ________

6) The arc length of the boundary of a surface is independent of the parametrization of the surface. 6) ________

7) The gradient of the divergence of the curl of a vector field is constantly zero. 7) ________

8) The points that satisfy $\theta = \pi/4$ and $\varphi = \pi/4$ in spherical coordinates form a surface which is part of a cone. 8) ________

9) For any vector field F and any curve r parametrized on [a, b] we have 9) ________

$$\int_a^b F(r(t)) \cdot r'(t)\, dt = F(r(b)) - F(r(a)).$$

10) The solid enclosed by the surfaces $z = 2 - \sqrt{x^2+y^2}$ and $z = \sqrt{x^2+y^2}$ has the volume 10) ________

$$\int_0^{2\pi} \int_0^1 \int_r^{2-r} dz\, dr\, d\theta \quad .$$

11) The line integral of $F(x,y) = (-y,x)$ along the (counterclockwise oriented) boundary of a region R is twice the area of R. 11) ________

SHORT QUESTIONS. (18 marks)

12) Check whether the vector field

$$\mathbf{F} = \left(\frac{10x^9y^{10}}{z^3}\right)\mathbf{i} + \left(\frac{10x^{10}y^9}{z^3}\right)\mathbf{j} - \left(\frac{3x^{10}y^{10}}{z^4}\right)\mathbf{k}$$

is conservative or not.

13) Find the potential function for the vector field

$$\mathbf{F} = \frac{1}{z}\mathbf{i} - 4\mathbf{j} - \frac{x}{z^2}\mathbf{k}.$$

Compute the path integral

$$\int_C \mathbf{F} \cdot d\mathbf{r}$$

for the curve C parametrized by (cos t, sin t, t) for $2\pi \leq t \leq 3\pi$.

14) Find a parametrization of the surface S which is the portion of the cone $\frac{x^2}{25} + \frac{y^2}{25} = \frac{z^2}{16}$ that lies between z = 2 and z = 3. Use it to find its surface area.

15) Find the path integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F} = -\sqrt{x^2 + y^2}\,\mathbf{i} + \sqrt{x^2 + y^2}\,\mathbf{j}$ and C is the (counterclockwise) boundary of the region defined by the polar coordinate inequalities $6 \le r \le 8$ and $0 \le \theta \le \pi$.

16) Find the flux $\int_S$ **curl F** $\cdot$ d**N** where S is the surface parametrized by $\mathbf{r}(r, \theta) = \left(r \cos \theta, r \sin \theta, \left(4 - r^2\right)\right)$ for $0 \le r \le 2$ and $0 \le \theta \le 2\pi$, and $\mathbf{F} = 3y\mathbf{i} + 5z\mathbf{j} - 2x\mathbf{k}$.

17) Find the flux $\int_S$ **F** $\cdot$ d**N** where the surface S is the boundary of the solid cube cut by the coordinate planes and the planes x = 1, y = 1, and z = 1, and $\mathbf{F} = \langle z, xy, zy \rangle$.