MA225C Mock Mid-Term Test 2

Name: ________________________________ (1 mark)

TRUE/FALSE. Write 'T' if the statement is true and 'F' if the statement is false. (10 marks)

1) The directional derivative of a function f(x,y) at a local minimum is positive in every direction. 1) _______

2) If r(t)=(x(t),y(t),z(t)) is a curve on the surface g(x,y,z) = 1, then $\nabla g(r(t)) \cdot r'(t) = 1$. 2) _______

3) If f(x, y) = g(x) is a function of x only, then the discriminant $\left(\frac{\partial^2 f}{\partial x^2}\right)\left(\frac{\partial^2 f}{\partial y^2}\right) - \left(\frac{\partial^2 f}{\partial x \partial y}\right)^2 = 0$ at every critical point. 3) _______

4) Given a function f(x, y) and a curve r(t)=(x(t),y(t)) which satisfies $r'(t) = \nabla f(r(t))$, then $\frac{d}{dt}(f(r(t))) \geq 0$. 4) _______

5) The integral $\int_R 1\, dxdy$ is the area of the region R in the xy-plane. 5) _______

6) If f(x, y) is a linear function in x, y, then $\nabla f(x, y)$ is independent of (x, y). 6) _______

7) The linearization of $f(x, y) = 4 + x^3 + y^3$ at (0, 0) is $L(x, y) = 4 + 3x^2 + 3y^2$. 7) _______

8) The length of the gradient of f at a critical point is positive if the discriminant $\left(\frac{\partial^2 f}{\partial x^2}\right)\left(\frac{\partial^2 f}{\partial y^2}\right) - \left(\frac{\partial^2 f}{\partial x \partial y}\right)^2$ is strictly positive. 8) _______

9) For any two functions f(x,y,z) and g(x,y,z) and any vector v we have $\nabla_v(f + g) = \nabla_v f + \nabla_v g$. 9) _______

10) The surfaces x + y + z = 0 and $x^2 + y^2 + z^2 - x - y - z = 0$ have the same tangent plane at (0, 0, 0). 10) _______

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question. (8 marks)

Solve the problem.

11) Evaluate $\frac{\partial u}{\partial y}$ at (x, y, z) = (4, 2, 0) for the function $u = e^{pq}\cos(r)$; $p = \frac{1}{x}$, $q = x^2 \ln y$, $r = z$. 11) _______

A) 64 B) 0 C) 2 D) 32

Find the absolute maxima and minima of the function on the given domain.

12) $f(x, y) = x^2 + 8x + y^2 + 14y + 8$ on the rectangular region $-1 \le x \le 1$, $-2 \le y \le 2$ 12) ______

A) Absolute maximum: 60 at (2, 2); absolute minimum: 8 at (0, 0)
B) Absolute maximum: 49 at (1, 2); absolute minimum: -23 at (-1, -2)
C) Absolute maximum: 49 at (1, 2); absolute minimum: 8 at (0, 0)
D) Absolute maximum: 60 at (2, 2); absolute minimum: -23 at (-1, -2)

Find the extreme values of the function subject to the given constraint.

13) $f(x, y, z) = x^3 + y^3 + z^3$, $x^2 + y^2 + z^2 = 4$ 13) ______

A) Maximum: 8 at (2, 0, 0), (0, 2, 0), (0, 0, 2); minimum: -8 at (-2, 0, 0), (0, -2, 0), (0, 0, -2)
B) Maximum: 8 at (2, 0, 0), (0, 2, 0), (0, 0, 2); minimum: 0 at (0, 0, 0)
C) Maximum: 8 at (2, 0, 0); minimum: -8 at (-2, 0, 0)
D) Maximum: 8 at (2, 0, 0); minimum: 0 at (0, 0, 0)

Integrate the function f over the given region.

14) $f(x, y) = xy$ over the triangular region with vertices (0, 0), (1, 0), and (0, 1) 14) ______

A) $\frac{1}{4}$ B) $\frac{1}{12}$ C) $\frac{1}{24}$ D) $\frac{1}{2}$

SHORT QUESTIONS. (6 marks)

15) Find the area of the region enclosed by the curve $r = 7 + \cos\theta$ specified by polar coordinates (r,θ) (for $0 \le \theta \le 2\pi$). (You may use the formula $\cos 2\theta = 2\cos^2\theta - 1$.)

16) Given a curve $r(t) = (t-t^3, 3t^2-3t, t)$ and a function $f(x,y,z)$. Suppose $\nabla f(0,0,0) = (2,-1,1)$. Find $\left.\frac{d}{dt}\right|_{t=0} f(r(t))$.