MA225D Mock Test 1

Name: _____

TRUE/FALSE. Write 'T' if the statement is true and 'F' if the statement is false. (1 mark each)

1) The vector $\langle 1/5, 2/5, 2/5 \rangle$ is a unit vector. 1) _____

2) Two vectors $\vec{v}$ and $\vec{w}$ are parallel if $\vec{v} \cdot \vec{w} = \vec{0}$. 2) _____

3) If two planes $ax + by + cz = d$ and $Ax + By + Cz = D$ are parallel, then $a = A, b = B$, and $c = C$. 3) _____

4) Every point on the parametric curve $r(t) = (t, t^2, -t)$ lies on the surface $xz + y = 0$. 4) _____

5) $\text{Proj}_{\vec{u}} \vec{v} = \text{Proj}_{\vec{v}} \vec{u}$ for all vectors $\vec{u}$ and $\vec{v}$. 5) _____

6) The curvature of the curves $r(t) = (t, t^2, t^3)$ and $R(t) = (t^2, t^4, t^6)$ are the same at $t=1$. 6) _____

7) The vector $\langle -5,4,1 \rangle$ is parallel to the plane $-5x + 4y + z = 2$. 7) _____

8) There are vectors $\vec{u}$ and $\vec{v}$ such that $\vec{u} \cdot \vec{v} = \|\vec{u} \times \vec{v}\|$. 8) _____

MULTIPLE CHOICE. (2 marks each)

The position vector of a particle is r(t). Find the requested vector.

9) The velocity at $t = 1$ for $r(t) = (2 - 4t^2)i + (6t + 5)j - e^{-6t}k$ 9) _____

A) $v(1) = 8i + 6j + 6e^{-6}k$ B) $v(1) = -8i + 6j + 6e^{-6}k$

C) $v(1) = -4i + 6j + 6e^{-6}k$ D) $v(1) = -8i + 6j - 6e^{-6}k$

10) What is the geometric object defined by $z = x^2 + y^2$? 10) _____

A) a circle B) a parabola C) a paraboloid D) a circular cone

11) What is the volume of the parallelepiped spanned by the vectors $\langle 1,0,0 \rangle, \langle 0,2,0 \rangle$ and $\langle 1,1,1 \rangle$? 11) _____

A) 1 B) 2 C) D) 0

12) The following equations each describe the motion of a particle. For which path is the particle's speed constant? 12) _____

(1) $r(t) = t^2 i + t^5 j$
(2) $r(t) = \cos(8t)i + \sin(5t)j$
(3) $r(t) = ti + tj$
(4) $r(t) = \cos(9t^2)i + \sin(9t^2)j$

A) Path (3)
B) Path (4) and Path (2)
C) Path (1)
D) Path (2) and Path (3)

The position vector of a particle is r(t). Find the requested vector.

13) The acceleration at t = 1 for $r(t) = (3t - 2t^4)i + (2 - t)j + (6t^2 - 7t)k$ 13) _____

A) $a(1) = 24i + 12k$
B) $a(1) = -6i + 12k$
C) $a(1) = -24i - j + 12k$
D) $a(1) = -24i + 12k$

The vector r(t) is the position vector of a particle at time t. Find the angle between the velocity and the acceleration vectors at time t = 0.

14) $r(t) = (4t^2 + 7)i + (3t^3 - 2t)k$ 14) _____

A) π
B) $\frac{\pi}{2}$
C) $\frac{\pi}{4}$
D) 0

For the smooth curve r(t), find the parametric equations for the line that is tangent to r at the given parameter value $t = t_0$.

15) $r(t) = (6 \sin t)i - (9 \cos 3t)j + e^{-10t}k$; $t_0 = 0$ 15) _____

A) $x = 6t, y = -9, z = 1 - 10t$
B) $x = 6t, y = 9, z = 1 + t$
C) $x = 6, y = -9t, z = -10 + t$
D) $x = 6t, y = -9, z = 1 - t$

SHORT QUESTIONS.

16) Write down a parametrization for the following. (8 marks)

a) $4x^2 + 9y^2 = 1$, $z = 3$.

b) $x^2 + y^2 = z^2 + 1$.

c) $x^2 + 2y^2 + 4z^2 = 4$.

d) The graph of the function $f(x,y) = e^y (\sin x)$.

e) The plane normal to (1,1,1) passing through the point (0,0,1).

17) Compute the first derivatives for the following. (4 marks)

a) $\gamma(t) = (\cos(e^{2t}), \sin(e^{2t}), e^{2t})$.

b) $(t,2t,3t^2) \cdot (\cos(t),\sin(t),\log(t))$ at $t=\pi$. (log(t)=ln(t) throughout this course.)

18) Find the arc length parameter of the curve (6 marks)

$\gamma(t) = (4\cos t, 4\sin t, 5t)$

by evaluating $s = \int_0^t |\gamma'(\tau)|\, d\tau$.

Compute $\frac{ds}{dt}$ and $\frac{dt}{ds}$. What is the physical meaning of $\frac{ds}{dt}$?