MA225D Mock Final Exam

Name: ________________________________ (2 marks)

TRUE/FALSE. Write 'T' if the statement is true and 'F' if the statement is false. (8 marks)

1) Let $f(x,y,z)$ be a function. The line integral of $\text{curl}(\nabla f)$ around any circle is zero. 1) _______

2) The flux of the curl of a vector field through the disc $x^2 + y^2 \le 1$, $z = 0$ is always zero. 2) _______

3) If a function $f(x,y)$ has a critical point at $(0,0)$ then $\text{div}(\text{grad}(f))(0,0)$ is zero. 3) _______

4) The gradient of the divergence of the curl of a vector field is constantly zero. 4) _______

5) The points in the 3d space that satisfy $\theta = \pi/4$ and $\varphi = \pi/4$ in spherical coordinates form a surface which is part of a cone. 5) _______

6) For any vector field F and any curve r parametrized on [a, b] we have 6) _______

$$\int_a^b F(r(t)) \cdot r'(t)\, dt = F(r(b)) - F(r(a)).$$

7) The solid enclosed by the surfaces $z = 2 - \sqrt{x^2+y^2}$ and $z = \sqrt{x^2+y^2}$ has the volume 7) _______

$$\int_0^{2\pi} \int_0^1 \int_r^{2-r} dz\, dr\, d\theta \quad .$$

8) In the plane, the line integral of $F(x,y) = (-y,x)$ along the (counterclockwise oriented) boundary of a region R is twice the area of R. 8) _______

SHORT QUESTIONS. (40 marks)

9) Check whether the vector field

$$F = \left(\frac{10x^9y^{10}}{z^3}\right)i + \left(\frac{10x^{10}y^9}{z^3}\right)j - \left(\frac{3x^{10}y^{10}}{z^4}\right)k$$

is a gradient vector field or not. (4 marks)

10) Find the potential function for the vector field

$F = \frac{1}{z} i - 4j - \frac{x}{z^2} k.$

Compute the path integral

$$\int_C F \cdot dC$$

for the curve C parametrized by (cos t, sin t, t) for $2\pi \le t \le 3\pi$. (6 marks)

11) Find a parametrization of the surface S which is the portion of the cone $\frac{x^2}{25} + \frac{y^2}{25} = \frac{z^2}{16}$ that lies between z = 2 and z = 3. Find the area element. Then find the surface area of S. (8 marks)

12) Find the path integral $\int_C F \cdot dC$, where $F = -\sqrt{x^2 + y^2}\,i + \sqrt{x^2 + y^2}\,j$ and C is the (counterclockwise) boundary of the region defined by the polar coordinate inequalities $6 \le r \le 8$ and $0 \le \theta \le \pi$. (8 marks)

13) Parametrize the boundary curve of the surface given by $r(r, \theta) = \left(r \cos \theta, r \sin \theta, \left(4 - r^2\right)\right)$ for $0 \le r \le 2$ and $0 \le \theta \le 2\pi$.

Then find the flux $\int_S \text{curl } F \cdot dS$ where $F = 3yi + 5zj - 2xk$.

(You can freely take your own choice of orientation.) (8 marks)

14) Find the flux integral $\int_S \mathbf{F} \cdot d\mathbf{S}$ where the surface S is the boundary of the solid cube cut by the coordinate planes x=0, y=0, z=0, and the planes x=1, y=1,z=1. S takes the outward orientation. $\mathbf{F} = \langle z, xy, zy \rangle$. (6 marks)