MA225D Mock Test 1

Name: _____

TRUE/FALSE. Write 'T' if the statement is true and 'F' if the statement is false. (1 mark each)

1) The vector $\langle 1/5, 2/5, 2/5\rangle$ is a unit vector. 1) _____

2) Two vectors $\vec{v}$ and $\vec{w}$ are parallel if $\vec{v} \cdot \vec{w} = \vec{0}$. 2) _____

3) If two planes $ax + by + cz = d$ and $Ax + By + Cz = D$ are parallel, then $a = A, b = B$, and $c = C$. 3) _____

4) Every point on the parametric curve $r(t) = (t, t^2, -t)$ lies on the surface $xz + y = 0$. 4) _____

5) $\text{Proj}_{\vec{u}} \vec{v} = \text{Proj}_{\vec{v}} \vec{u}$ for all vectors $\vec{u}$ and $\vec{v}$. 5) _____

6) The curvature of the curves $r(t) = (t, t^2, t^3)$ and $R(t) = (t^2, t^4, t^6)$ are the same at $t=1$. 6) _____

7) The vector $\langle -5,4,1\rangle$ is parallel to the plane $-5x + 4y + z = 2$. 7) _____

8) There are vectors $\vec{u}$ and $\vec{v}$ such that $\vec{u} \cdot \vec{v} = \|\vec{u} \times \vec{v}\|$. 8) _____

MULTIPLE CHOICE. (2 marks each)

The position vector of a particle is r(t). Find the requested vector.

9) The velocity at $t = 1$ for $r(t) = (2 - 4t^2)i + (6t + 5)j - e^{-6t}k$ 9) _____

A) $v(1) = 8i + 6j + 6e^{-6}k$ B) $v(1) = -8i + 6j + 6e^{-6}k$

C) $v(1) = -4i + 6j + 6e^{-6}k$ D) $v(1) = -8i + 6j - 6e^{-6}k$

10) What is the geometric object defined by $z = x^2 + y^2$? 10) _____

A) a circle B) a parabola C) a paraboloid D) a circular cone

11) What is the volume of the parallelepiped spanned by the vectors $\langle 1,0,0\rangle$, $\langle 0,2,0\rangle$ and $\langle 1,1,1\rangle$? 11) _____

A) 1 B) 2 C) D) 0

The position vector of a particle is r(t). Find the requested vector.

12) The acceleration at t = 1 for $r(t) = (3t - 2t^4)i + (2 - t)j + (6t^2 - 7t)k$ 12) _____

A) a(1) = 24i + 12k
B) a(1) = -6i + 12k
C) a(1) = -24i - j + 12k
D) a(1) = -24i + 12k

For the smooth curve r(t), find the parametric equations for the line that is tangent to r at the given parameter value $t = t_0$.

13) $r(t) = (6 \sin t)i - (9 \cos 3t)j + e^{-10t}k$; $t_0 = 0$ 13) _____

A) x = 6t, y = -9, z = 1 - 10t
B) x = 6t, y = 9, z = 1 + t
C) x = 6, y = -9t, z = -10 + t
D) x = 6t, y = -9, z = 1 - t

SHORT QUESTIONS.

14) Write down a parametrization for the following. (10 marks)

a) $4x^2 + 9y^2 = 1$, z = 3.

b) $x^2 + y^2 = z^2 + 1$.

c) $x^2 + 2y^2 + 4z^2 = 4$.

d) The graph of the function $f(x,y) = e^y (\sin x)$.

e) The plane normal to (1,1,1) passing through the point (0,0,1).

15) Compute the first derivatives for the following. (6 marks)

a) $\gamma(t) = (\cos(e^{2t}), \sin(e^{2t}), e^{2t})$.

b) $(t,2t,3t^2)\cdot(\cos(t),\sin(t),\log(t))$ at $t=\pi$. ($\log(t)=\ln(t)$ throughout this course.)

16) Find the arc length parameter of the curve (6 marks)

$\gamma(t) = (4\cos t, 4\sin t, 5t)$

by evaluating $s = \int_0^t |\gamma'(\tau)|\, d\tau$.

Compute $\frac{ds}{dt}$ and $\frac{dt}{ds}$. What is the physical meaning of $\frac{ds}{dt}$?

1) FALSE
2) FALSE
3) FALSE
4) TRUE
5) FALSE
6) TRUE
7) FALSE
8) TRUE
9) B
10) C
11) B
12) D
13) A
14) a) $(\frac{1}{2}\cos t, \frac{1}{3}\sin t, 3)$.

b) $(x,y,z) = (\sqrt{1+u^2}\cos v, \sqrt{1+u^2}\sin v, u)$. Or $(\cosh(u)\cos(\theta),\cosh(u)\sin(\theta),\sinh(u))$.

c) $(x,y,z) = (2\sin(\varphi)\cos(\theta),\sqrt{2}\sin(\varphi)\sin\theta,\cos\varphi)$.

d) $(x,y,e^v (\sin u))$.

e) $(x,y,z) = (u,v,1-u-v)$.

15) a) $2\, e^{2t}\, (-\sin(e^{2t}), \cos(e^{2t}),1)$.

b) $-1+\pi+6\pi \log(\pi)$.

16) $s = \sqrt{41}\, t$.

$\frac{ds}{dt} = \sqrt{41}$.

$\frac{dt}{ds} = \frac{1}{\sqrt{41}}$.