MA225B Mock Mid-Term Test 2

Name: ___________________________________

TRUE/FALSE. Write 'T' if the statement is true and 'F' if the statement is false. (8 marks)

1) The directional derivative of a function f(x,y) at a local minimum is positive in every direction. 1) ________

2) If r(t)=(x(t),y(t),z(t)) is a curve on the surface g(x,y,z) = 1, then $\nabla g(r(t)) \cdot r'(t) = 1$. 2) ________

3) If f(x, y) = g(x) is a function of x only, then the discriminant $\left(\frac{\partial^2 f}{\partial x^2}\right)\left(\frac{\partial^2 f}{\partial y^2}\right) - \left(\frac{\partial^2 f}{\partial x \partial y}\right)^2 = 0$ at every critical point. 3) ________

4) Given a function f(x, y) and a curve r(t)=(x(t),y(t)) which satisfies $r'(t) = \nabla f|_{r(t)}$, then $\frac{d}{dt}(f(r(t))) \geq 0$. 4) ________

5) If f(x,y) is a linear function in x, y, then $\nabla f(x,y)$ is independent of (x,y). 5) ________

6) $\|\nabla f\|$ at a critical point is positive if the discriminant $\left(\frac{\partial^2 f}{\partial x^2}\right)\left(\frac{\partial^2 f}{\partial y^2}\right) - \left(\frac{\partial^2 f}{\partial x \partial y}\right)^2$ is strictly positive. 6) ________

7) For any two functions f(x,y,z) and g(x,y,z) and any vector v we have $\nabla_v(f + g) = \nabla_v f + \nabla_v g$. 7) ________

8) The surfaces x + y + z = 0 and $x^2 + y^2 + z^2 - x - y - z = 0$ have the same tangent plane at (0, 0, 0). 8) ________

MULTIPLE CHOICE. (14 marks)

9) Evaluate $\frac{\partial u}{\partial y}$ at (x, y, z) = (4, 2, 0) for the function $u = e^{pq}\cos(r)$; $p = \frac{1}{x}$, $q = x^2 \ln y$, r = z. 9) ________

A) 64 B) 0 C) 2 D) 32

10) Find the linearization of $f(x,y) = 4 + x^3 + y^3$ at (0, 0). 10) ______

A) $L(x,y) = 4 + 3x^2 + 3y^2$. B) L(x,y) = 4.

C) L(x,y) = 4 + x + y. D) $L(x,y) = 4 + x^3 + y^3$.

11) Find the tangent plane of the surface $x^2y^2z^2 = 1$ at the point (x,y,z) = (1,1,1). 11) ______

A) $x^2y^2z^2 = 0$. B) x+y+z=1. C) x+y+z = 3. D) x=y=z=1.

12) Find the absolute maxima and minima of $f(x, y) = x^2 + 8x + y^2 + 14y + 8$ on the rectangular region $-1 \leq x \leq 1$, $-2 \leq y \leq 2$ 12) ______

A) Absolute maximum: 60 at (2, 2); absolute minimum: 8 at (0, 0)
B) Absolute maximum: 49 at (1, 2); absolute minimum: -23 at (-1, -2)
C) Absolute maximum: 49 at (1, 2); absolute minimum: 8 at (0, 0)
D) Absolute maximum: 60 at (2, 2); absolute minimum: -23 at (-1, -2)

13) Find $\partial_v f(-1,1)$ for f(x,y) = cos(xy) and v = (2,1). 13) ______

A) sin(1). B) -sin(1). C) 0. D) (sin(1),-sin(1)).

14) Find $\frac{\partial w}{\partial r}$ when r = -2 and s = -1 if w(x, y, z) = xz + y^2, x = 4r + 2, y = r + s, and z = r - s. 14) ______

A) $\frac{\partial w}{\partial r} = -4$ B) $\frac{\partial w}{\partial r} = -16$ C) $\frac{\partial w}{\partial r} = -10$ D) $\frac{\partial w}{\partial r} = -7$

Find all the local maxima, local minima, and saddle points of the function.

15) $f(x, y) = 2xy + 2x + 6y$ 15) ______

A) $f(-3, 1) = -6$, local minimum; $f(3, -1) = -6$, local minimum
B) $f(-3, 1) = -6$, saddle point; $f(3, -1) = -6$, saddle point
C) $f(3, 1) = 18$, local maximum
D) $f(-3, -1) = -6$, saddle point

SHORT QUESTIONS.

16) Given a curve $r(t) = (t-t^3, 3t^2-3t, t)$ and a function $f(x,y,z)$. Suppose $\nabla f(0,0,0) = (2,-1,1)$. Find $\left.\frac{d}{dt}\right|_{t=0} f(r(t))$.

(5 marks)

17) Suppose $\cos xy + x^3 = y^3$. Find dy/dx. (5 marks)

18) Find the global maximum and mininum values of the function (8 marks)

$f(x, y, z) = x^3 + y^3 + z^3$

subject to the constraint

$x^2 + y^2 + z^2 = 4.$

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19) Let $f(x,y) = 2xy^2 + 4x^2 - 3xy - 7x + 2y + 9$.

Find $f(0,0)$, $\partial_x f(0,0)$, $\partial_y f(0,0)$, $\partial^2_{xx} f(0,0)$, $\partial^2_{xy} f(0,0)$, $\partial^2_{yy} f(0,0)$, $\partial^3_{xyy} f(0,0)$.

What are their relations with the coefficients of $f(x,y)$?