MA225B Mock Mid-Term Test 2

Name: _________________________________

TRUE/FALSE. Write 'T' if the statement is true and 'F' if the statement is false. (8 marks)

1) The directional derivative of a function \( f(x,y) \) at a local minimum is positive in every direction. 1) _____

2) If \( r(t) = (x(t), y(t), z(t)) \) is a curve on the surface \( g(x,y,z) = 1 \), then \( \nabla g(r(t)) \cdot r'(t) = 1 \). 2) _____

3) If \( f(x, y) = g(x) \) is a function of \( x \) only, then the discriminant \( \begin{vmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial y^2} \\ \frac{\partial^2 f}{\partial y^2} & \frac{\partial^2 f}{\partial x \partial y} \end{vmatrix} \) = 0 at every critical point. 3) _____

4) Given a function \( f(x, y) \) and a curve \( r(t) = (x(t), y(t)) \) which satisfies \( r'(t) = \nabla f|_{r(t)} \), then \( \frac{d}{dt} (f(r(t))) \geq 0 \). 4) _____

5) If \( f(x,y) \) is a linear function in \( x, y \), then \( \nabla f(x,y) \) is independent of \( (x,y) \). 5) _____

6) \( \| \nabla f \| \) at a critical point is positive if the discriminant \( \begin{vmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial y^2} \\ \frac{\partial^2 f}{\partial y^2} & \frac{\partial^2 f}{\partial x \partial y} \end{vmatrix} \) is strictly positive. 6) _____

7) For any two functions \( f(x,y,z) \) and \( g(x,y,z) \) and any vector \( v \) we have \( \nabla_v (f + g) = \nabla_v f + \nabla_v g \). 7) _____

8) The surfaces \( x + y + z = 0 \) and \( x^2 + y^2 + z^2 = x - y - z = 0 \) have the same tangent plane at \((0,0,0)\). 8) _____

MULTIPLE CHOICE. (14 marks)

9) Evaluate \( \frac{\partial u}{\partial y} \) at \((x, y, z) = (4, 2, 0)\) for the function \( u = e^p q \cos(r) ; \ p = \frac{1}{x} , \ q = x^2 \ln y , \ r = z \). 9) _____

A) 64 \hspace{1cm} B) 0 \hspace{1cm} C) 2 \hspace{1cm} D) 32

10) Find the linearization of \( f(x,y) = 4 + x^3 + y^3 \) at \((0, 0)\). 10) _____

A) \( L(x,y) = 4 + 3x^2 + 3y^2 \). \hspace{1cm} B) \( L(x,y) = 4 \).

C) \( L(x,y) = 4 + x + y \). \hspace{1cm} D) \( L(x,y) = 4 + x^3 + y^3 \).
11) Find the tangent plane of the surface \( x^2y^2z^2 = 1 \) at the point \((x,y,z) = (1,1,1)\).

A) \( x^2y^2z^2 = 0 \).
B) \( x+y+2z = 1 \).
C) \( x+z = 3 \).
D) \( x+y+z = 1 \).

12) Find the absolute maxima and minima of \( f(x,y) = x^2 + 8x + y^2 + 14y + 8 \) on the rectangular region \(-1 \leq x \leq 1, -2 \leq y \leq 2\).

A) Absolute maximum: 60 at \((2, 2)\); absolute minimum: 8 at \((0, 0)\).
B) Absolute maximum: 49 at \((1, 2)\); absolute minimum: -23 at \((-1, -2)\).
C) Absolute maximum: 49 at \((1, 2)\); absolute minimum: 8 at \((0, 0)\).
D) Absolute maximum: 60 at \((2, 2)\); absolute minimum: -23 at \((-1, -2)\).

13) Find \( \delta_v f \) \((-1,1)\) for \( f(x,y) = \cos(xy) \) and \( v = (2,1) \).

A) \( \sin(1) \).
B) \( -\sin(1) \).
C) 0.
D) \( (\sin(1), -\sin(1)) \).

14) Find \( \frac{\partial w}{\partial r} \) when \( r = -2 \) and \( s = -1 \) if \( w(x,y,z) = xz^2 + y^2, x = 4r + 2, y = r + s, \) and \( z = r - s \).

A) \( \frac{\partial w}{\partial r} = 4 \)
B) \( \frac{\partial w}{\partial r} = -16 \)
C) \( \frac{\partial w}{\partial r} = -10 \)
D) \( \frac{\partial w}{\partial r} = -7 \)

Find all the local maxima, local minima, and saddle points of the function.

15) \( f(x,y) = 2xy + 2x + 6y \)

A) \( f(-3,1) = -6 \), local minimum; \( f(3,-1) = -6 \), local minimum
B) \( f(-3,1) = -6 \), saddle point; \( f(3,-1) = -6 \), saddle point
C) \( f(3,1) = 18 \), local maximum
D) \( f(-3,-1) = -6 \), saddle point
SHORT QUESTIONS.

16) Given a curve \( r(t) = (t- t^3, 3t^2 - 3t, t) \) and a function \( f(x, y, z) \). Suppose \( \nabla f \left( 0,0,0 \right) = (2, 1, 1) \). Find \( \frac{d}{dt} \bigg|_{t=0} f(r(t)) \).

(5 marks)

17) Suppose \( \cos xy + x^3 = y^3 \). Find \( \frac{dy}{dx} \).

(5 marks)
18) **Find the global maximum and minimum values of the function** (8 marks)

\[ f(x, y, z) = x^3 + y^3 + z^3 \]

**subject to the constraint**

\[ x^2 + y^2 + z^2 = 4. \]
19) Let \( f(x,y) = 2x^2 + 4xy - 3x + 2y + 9 \).

Find \( f(0,0) \), \( \partial_x f(0,0) \), \( \partial_y f(0,0) \), \( \partial^2_{xx} f(0,0) \), \( \partial^2_{xy} f(0,0) \), \( \partial^2_{yy} f(0,0) \), \( \partial^3_{xxy} f(0,0) \).

What are their relations with the coefficients of \( f(x,y) \)?