# Homework: Surface area

by Oliver Knill

1 Find the surface area of the surface given by

$$z = \frac{2}{3}(x^{3/2} + y^{3/2}),\ 0 \leq x \leq 1,\ 0 \leq y \leq 1 .$$

2 Find the area of the surface given by the **helicoid**

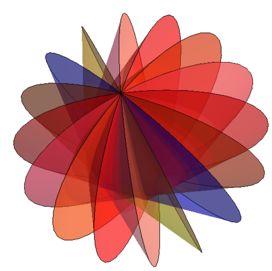
$$\vec{r}(u,v) = \langle u\cos(v), u\sin(v), v\rangle .$$

with $0 \leq u \leq 1,\ 0 \leq v \leq \pi$.

3 A decorative paper lantern is made of 8 surfaces. Each is parametrized by

$$\vec{r}(t,z) = \langle 10z\cos(t), 10z\sin(t), z\rangle$$

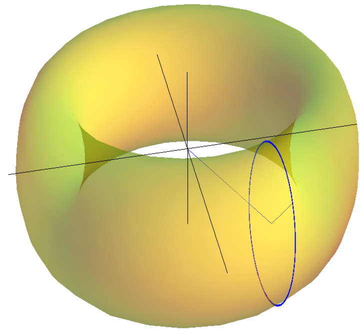
with $0 \leq t \leq 2\pi$ and $0 \leq z \leq 1$ and then translated or rotated. Find the total surface area of the lantern.



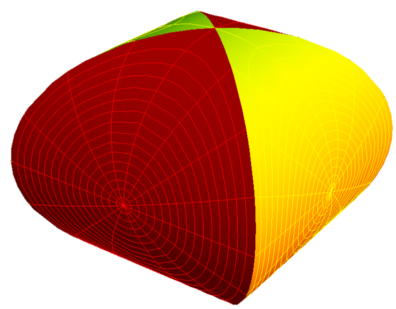
4 The figure shows the torus obtained by rotating about the $z$-axis the circle in the $xz$-plane with center $(b,0,0)$ and radius $a < b$. Parametric equations for the torus are

$$\begin{aligned} x &= b\cos\theta + a\cos\alpha\cos\theta \\ y &= b\sin\theta + a\cos\alpha\sin\theta \\ z &= a\sin\alpha\ , \end{aligned}$$

where $\theta$ and $\alpha$ are the angles shown in the figure. Find the surface area of the torus.



5 The volume and surface area of the solid obtained by intersecting the solid cylinder $y^2+z^2 \leq 1$ with the solid cylinder $x^2+z^2 \leq 1$ has been found by Archimedes already. Find the surface area of the surface $S$ bounding this solid.



## Main definitions:

A surface $\vec{r}(u,v)$ parametrized on a parameter domain $R$ has the **surface area**

$$\int\int_R |\vec{r}_u(u,v) \times \vec{r}_v(u,v)| \; dudv \; .$$

## Examples:

| $\vec{r}(u,v)$ | $\lvert\vec{r}_u \times \vec{r}_v\rvert$ |
|---|---|
| $\langle \rho\cos(u)\sin(v), \rho\sin(u)\sin(v), \rho\cos(v)\rangle$ | $\rho^2\lvert\sin(v)\rvert$ |
| $\langle u, v, f(u,v)\rangle$ | $\sqrt{1+f_u^2+f_v^2}$ |
| $\langle f(v)\cos(u), f(v)\sin(u), v\rangle$ | $f(v)\sqrt{1+f'(v)^2}$ |