DIFFERENTIAL GEOMETRY HOMEWORK 12

LECTURER: SIU-CHEONG LAU

(1) Consider the triangle in the first octant $\{x, y, z \geq 0\}$ of the unit sphere $\{x^2 + y^2 + z^2 = 1\}$ bounded by the following three geodesics.

$$\gamma_1 = (\cos t, \sin t, 0).$$
$$\gamma_2 = (\cos t, \frac{\sqrt{2}}{2}\sin t, \frac{\sqrt{2}}{2}\sin t).$$
$$\gamma_3 = (0, \cos t, \sin t).$$

Find the area of the triangle using the Gauss-Bonnet formula. (Recall that the sphere has constant curvature $K = 1$.)

(2) Consider the triangle in the hyperbolic upper half plane bounded by the following three geodesics.

$$\gamma_1 = (\cos t, \sin t).$$
$$\gamma_2 = (\cos t + 1, \sin t).$$
$$\gamma_3 = (1/4, t).$$

Find the area of the triangle using the Gauss-Bonnet formula. (Recall that the hyperbolic upper half plane $\{y \geq 0\}$ has the first fundamental form $\frac{1}{y^2}(dx^2 + dy^2)$, and it has constant curvature -1.)

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