DIFFERENTIAL GEOMETRY HOMEWORK 4

LECTURER: SIU-CHEONG LAU

(1) Compute the Frenet frame and curvatures at $t = 0$ for the curve (t, t^2, t^3, t^4) in $\mathbb{R}^4$. Show that at $t = 0$,

$$\det\left(\left.\frac{dc}{ds}\right|_{t=0}, \left.\frac{d^2c}{ds^2}\right|_{t=0}, \left.\frac{d^3c}{ds^3}\right|_{t=0}, \left.\frac{d^4c}{ds^4}\right|_{t=0}\right) = \prod_{i=1}^{3}(\kappa_i(0))^{4-i}$$

where κ_i for $i = 1, \ldots, 3$ are the curvatures. (Feel free to use computer to help if you like.) BONUS: show that this is true for general Frenet curves.

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