

DIFFERENTIAL GEOMETRY HOMEWORK 5

LECTURER: SIU-CHEONG LAU

- (1) Consider the parametrized three-sphere $\mathbb{S}^3 \subset \mathbb{R}^4$ given by

$$f(\phi, \psi, \theta) = (\cos \phi \cos \psi \cos \theta, \sin \phi \cos \psi \cos \theta, \sin \psi \cos \theta, \sin \theta).$$

Find all (ϕ, ψ, θ) where df has the maximal rank.

- (2) Consider the following parametrization of the sphere \mathbb{S}^2

$$f(u, \phi) = \frac{1}{\cosh u} (\cos \phi, \sin \phi, \sinh u).$$

Show that df is angle-preserving, namely

$$\frac{\langle v, w \rangle}{\|v\| \|w\|} = \frac{\langle df(v), df(w) \rangle}{\|df(v)\| \|df(w)\|}.$$

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