DIFFERENTIAL GEOMETRY HOMEWORK 5

LECTURER: SIU-CHEONG LAU

Consider the torus T parametrized by

$$f(u,v) = ((a + b\cos u)\cos v, (a + b\cos u)\sin v, b\sin u)$$

where $a > b > 0$ are fixed.

(1) Compute its Gauss curvature, which is a function $K(u,v)$ on the torus. Where is $K(u,v) > 0, = 0, < 0$?
(2) Compute its mean curvature, which is a function $H(u,v)$ on the torus.
(3) (**Bonus**) Compute its total mean curvature $\int_T H^2\, dA$ where dA is the area form. Show that the total mean curvature achieves minimum at $a = \sqrt{2}b$.

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