Consider the torus $T$ parametrized by
\[ f(u, v) = ((a + b \cos u) \cos v, (a + b \cos u) \sin v, b \sin u) \]
where $a > b > 0$ are fixed.

(1) Compute its Gauss curvature, which is a function $K(u, v)$ on the torus. Where is $K(u, v) > 0$, $= 0$, $< 0$?

(2) Compute its mean curvature, which is a function $H(u, v)$ on the torus.

(3) (Bonus) Compute its total mean curvature $\int_T H^2 \, dA$ where $dA$ is the area form. Show that the total mean curvature achieves minimum at $a = \sqrt{2}b$. 

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