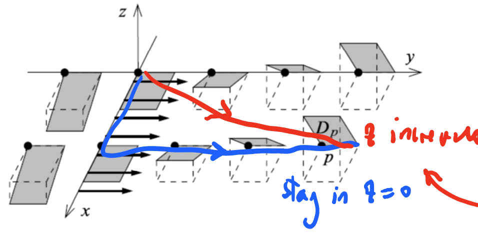
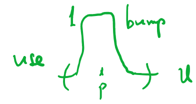
Distribution

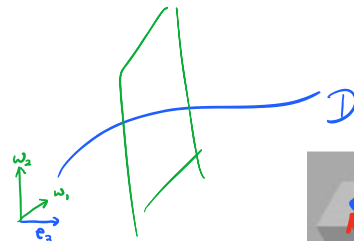
Monday, December 28, 2015 7:14 PM

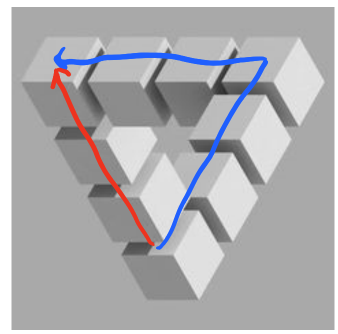
- **Distribution** D: subbundle of TM
- **(Local) integral manifold:** N with $TN = D|_N$ (Don't need embedded globally)
- **Examples**
  - Vector field and its integral curve
  - Spheres: $\langle \sum x_i \partial_i \rangle^\perp$
  - Not **integrable**: $\langle X = \partial_x + y\partial_z, Y = \partial_y \rangle$
- Necessary: **involutive** $[X, Y] \in \Gamma_{loc}(D)$.
- Involutive <=> $\Gamma(D)$ is Lie subalgebra
- integrable => involutive
- In local frames: $[e_i, e_j] \in \Gamma_{loc}(D)$.
- D in local one-forms: $Ker\ \omega_{k+1} \cap \cdots \cap Ker\ \omega_n$



Note:
$\partial_y \in D$
$\rightsquigarrow \varphi_t : \mathbb{R}^3 \circlearrowleft$
$(\varphi_t)_*$ does not preserve D!







- Analog: submanifold $S = \{u_{k+1} = \cdots = u_n = 0\}$ <-> defining ideal $C^\infty(M) \cdot \langle u_{k+1}, \dots, u_n \rangle = \{u|_S = 0\}$.
- P-form annihilates $D \Leftrightarrow \sum_i \omega_i \wedge \beta_i$ for some β_i.
  Hence locally {ann. forms} $= \Omega(M) \cdot \{\omega_{k+1}, \dots, \omega_n\}$.
  Integrability: want $\omega_i = df_i$ up to a factor μ.
- Involutive <=> {ann. forms} is closed under d
  Called differential ideal. Use $d\eta(X, Y) = X \cdot \eta(Y) - Y \cdot \eta(X) - \eta([X, Y])$.
- Need to talk about sheaf in the holomorphic setting
- Prob. 19.2

Distribution $D \subseteq_{\text{sub-bdl}} TM$	$C^\infty(M)$-module $\Gamma(D) \underset{loc.}{=} C^\infty(M) \cdot \{X_i : i=1..k\}$	annihilation ideal $\subset \Omega^\cdot(M)$ $Ann(D) \underset{loc.}{=} \Omega^\cdot(M) \cdot \{\omega_{k+1}, \dots, \omega_n\}$
involutive	closed under [,]	closed under d
integrable	$\Gamma(D) \underset{loc.}{=} C^\infty(M)\{\frac{\partial}{\partial u_i} : i=1..k\}$	$Ann(D) = \Omega^\cdot(M) \cdot \{du_i : i=k+1..n\}$

Motivation:
Vector field can always integrate to a local flow $\exp tX$.
Can we do this for several (independent) vector fields?