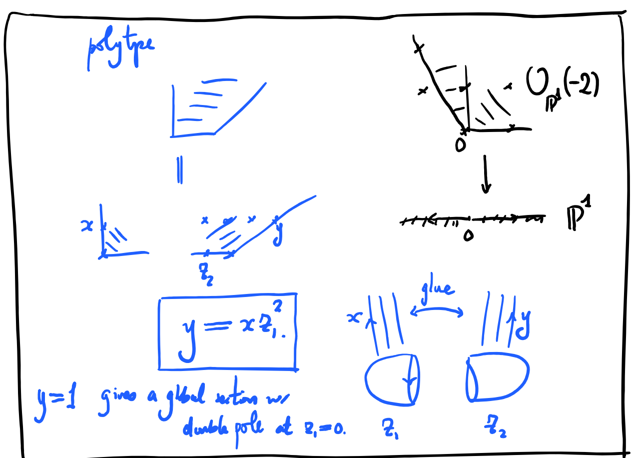
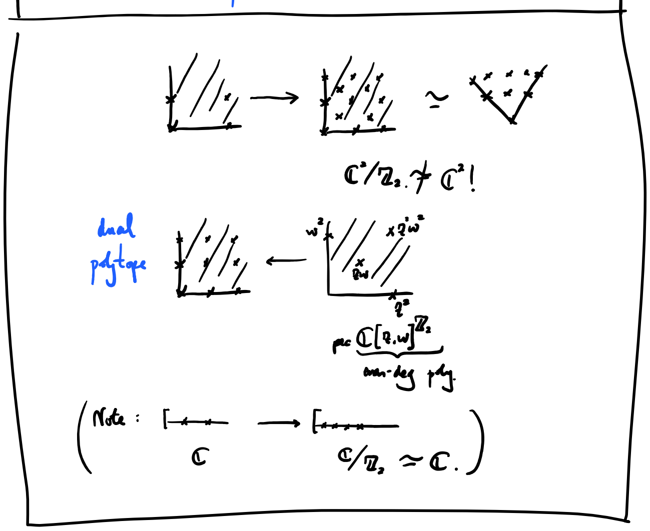
Toric morphisms

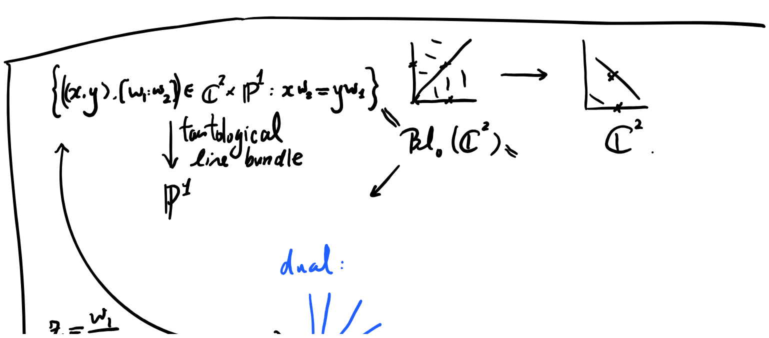
Monday, January 11, 2016 3:28 PM

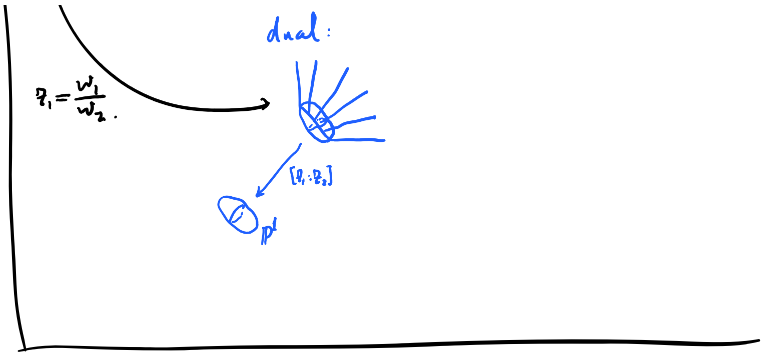
- Fan <-> polytope. The fan indicates how to compactify $\mathbb{C}^{\times n}$.
- Fan morphisms. $f:(N_1,\Sigma_1)\to(N_2,\Sigma_2), f(\sigma)\subset\tau\in\Sigma_2$. So $\mathbb{C}[\sigma^\vee\cap M_1]\leftarrow\mathbb{C}[\tau^\vee\cap M_2]$.
- ex. Refine the lattice: $\mathbb{C}^2\to\mathbb{C}^2/\mathbb{Z}_2$.
- ex. Blow-up of $\mathbb{C}^2$. Blow-up at a fixed point <-> "add a ray"
- $O_{\mathbb{P}^1}(k)$.
- A_n, A_∞
- Toric compactification (Hirzebruch surface) $\mathbb{F}_k$ of $O_{\mathbb{P}^1}(-k)$: $\mathbb{P}^1$-bundle.
- $O_{\mathbb{P}^1}(-1,-1)\to$ conifold, also $\to\mathbb{P}^1$. Flop.

- Compact <-> complete fan (every direction is compactified)
- Proper morphism <-> $f^{-1}(|\Sigma|)=|\Sigma'|$.
- Torus action: for $v\in N_{\mathbb{C}}/N\cong\mathbb{C}^{\times n}, e^{2\pi i v}\cdot z^\nu := \exp 2\pi i(v,\nu)\cdot z^\nu$;
  $e^{2\pi i v}\cdot z^{\nu_1+\nu_2}=\left(e^{2\pi i v}\cdot z^{\nu_1}\right)\left(e^{2\pi i v}\cdot z^{\nu_2}\right)$.
- Toric orbits $O_\tau := \{z^\nu=0 \Leftrightarrow \nu\in(\tau^\vee\cap N)-\tau^\perp\}\subset U_\tau=\operatorname{Spec}\mathbb{C}[z^\nu:\ \nu\in\tau^\vee\cap N]$
- ex: Fan for blow-up of $\mathbb{C}^3$.







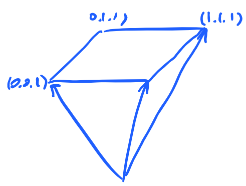


$\mathcal{O}_{\mathbb{P}^1}(-1) \oplus \mathcal{O}_{\mathbb{P}^1}(-1)$

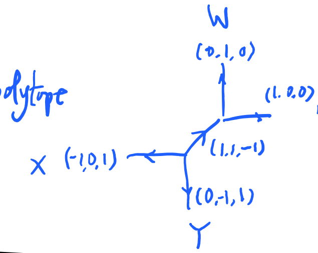
$\downarrow$ blow-up

conifold

fan



polytope



conifold:

$\mathrm{Spec}\left(\mathbb{C}[X,Y,Z,W] / \langle XZ - YW \rangle\right)$.

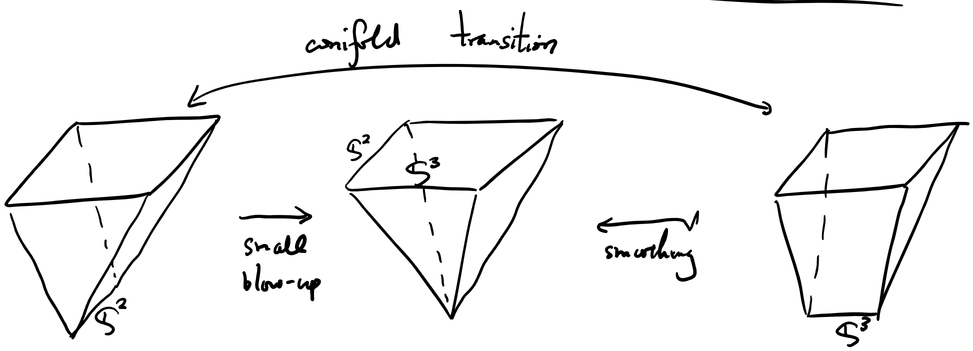
$\updownarrow$ smoothing

$\{XZ - YW = \varepsilon\} \subset \mathbb{C}^4$.

$\wr\!\!\mid$

T^*S^3.

(not toric!)



$\mathcal{O}_{\mathbb{P}^1}(-1) \oplus \mathcal{O}_{\mathbb{P}^1}(-1)$ conifold T^*S^3.