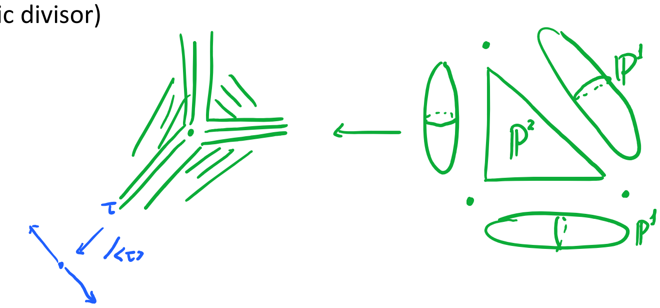
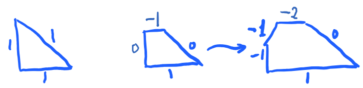
- Simply-connected if fan has top dimensional cone
- Euler characteristic = # top-dim cones
- Toric subvarieties $\overline{O_\tau} = \{z^\nu = 0\ \forall \nu \in \tau^\vee - \tau^\perp\}$ (itself toric with fan $(N, \Sigma_\tau)/(\tau)$); generate cohomology. (Can always use $\mathbb{C}^\times$ action to push any non-top-dim. cycle to be contained in a toric divisor)
- Weil divisor: formal sum of codimension-one subvarieties.
- Toric Weil divisors <-> rays of fan. Generates cohomology.
- $\nu \in M$ <--> toric meromorphic functions $\exp 2\pi i\,(\nu,\cdot)$ on $N_{\mathbb{C}}/N$.
- Principle divisors $(z^\nu) = \sum_i (\nu, v_i) D_i$; linear equivalence.
- Exact sequence $0 \to M \to \mathbb{Z}\langle D_1, \ldots D_m\rangle \to \mathbb{Z}\langle D_1, \ldots D_m\rangle/\text{lin. equiv.} \to 0$.
- Computing intersection numbers and cup product, ex. Hirzebruch surface $\mathbb{F}_n$
- Cech cohomology by toric charts. Fact: $H^{>0}(F) = 0\ \forall O_X$-sheaf F over affine var.
- Ex. compute the self-intersection numbers of toric curves in blow-ups of surfaces.

$\dot{0} \longleftrightarrow (\mathbb{C}^\times)^n$ not simply conn





Čech: $\mathcal{F}$: sheaf of gp.

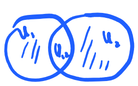
$\mathcal{U} = \{U_i\}$ open cover.

$\ni \{s_I \in \mathcal{F}(U_I) : |I| = k+1\} \triangleq s.$

w/ $s_I = \mathrm{sgn}(\pi)\, s_{\pi(I)}$ $\forall$ permut. π

$\downarrow$

$C^{k+1}(F) \ni \delta\cdot s$ on $U_J = \sum_{i=0}^{k+1} s_{(j_0 \ldots \hat{j_i} \ldots j_k)}\big|_{U_J}$.



$\xrightarrow{\text{coho.}} \check{H}(\mathcal{U}, F)$.

$\check{H}(F) \triangleq \varinjlim_{\mathcal{U}} \check{H}(\mathcal{U}, F)$.

(need refinement of $\mathcal{U}$.)

sheaf: open set $\longrightarrow$ group (of loc. sect.)

have restriction $F_U \to F_V$ for $U \supset V$.

(= id when $U = V$; respect composition)

loc. determine global:

$$\begin{cases} s|_{U_i} = t|_{U_i}\ \forall i,\ \bigcup_i U_i = U \Rightarrow s = t \text{ on } U. \\ s_{U_i} \text{ w/ } s_{U_i}|_{U_i \cap U_j} = s_{U_j}|_{U_i \cap U_j} \Rightarrow s \text{ on } \bigcup U_i. \end{cases}$$

e.g. $\Omega^\cdot, \mathcal{O}, \mathcal{O}(E), \mathbb{Z}, \mathbb{R}$.

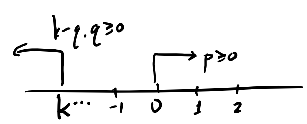
e.g. $\mathcal{O}(k)$. $(e_1 = z_2^k e_2.)$

U_1 U_2

$\mathbb{C}$ $\mathbb{C}$

$H^0 = \{\text{global sections}\} = \mathbb{C}^{\max\{k,0\}}$.

$H^1 = \{\text{sections on } U_{12}\} / \underbrace{\langle \sigma_1 - \sigma_2 \rangle}_{z_1^p e_1,\ z_2^q e_2}$ $\quad p, q \geq 0$



if $k \geq 0$, $z^\ell \sim 0\ \forall \ell \Rightarrow H^1 = 0$.

if $k < 0$, $z_1^{-1}, \ldots, z_1^{k+1}$ remained.

$\therefore H^1(\mathcal{O}(-k)) \simeq H^0(\mathcal{O}(k-2))$

(Serre duality: $H^p(\mathcal{F}) = (H^{n-p}(K \otimes \mathcal{F}^*))^*$.)

(Note: $H^0(\mathcal{O}(-1)) = H^1(\mathcal{O}(-1)) = 0$!)