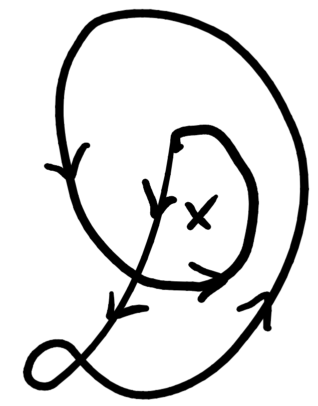
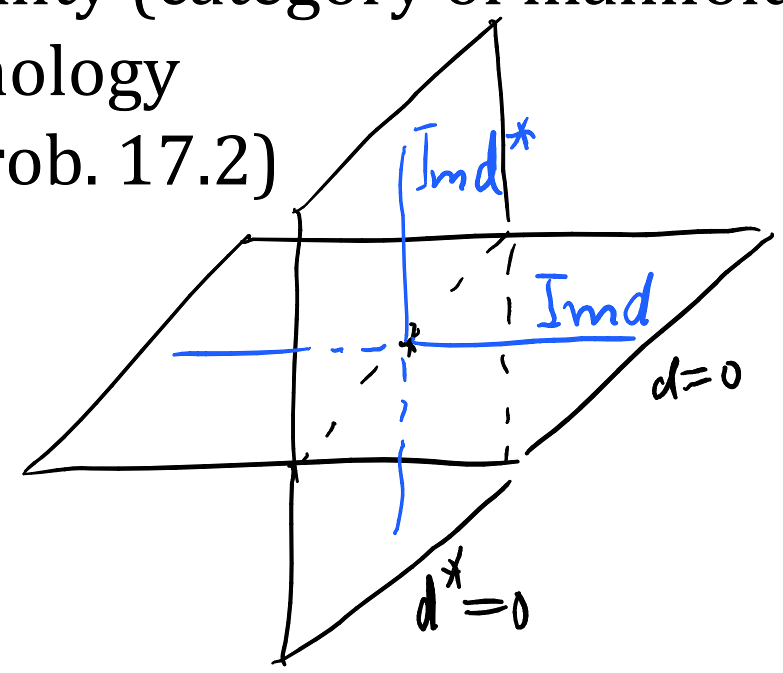
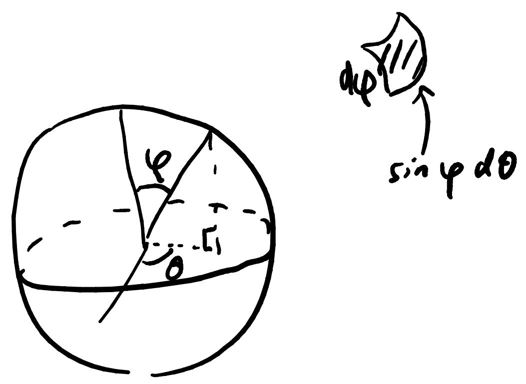
$$\oint \frac{dz}{\underbrace{2\pi i z}_{\text{cl, not exact.}}} = \text{winding \# around } 0.$$



- Motivation: angle 1-form and winding numbers
- Recall Differential forms (p-volumes), wedge (linear algebra)
- Differential; De Rham complex; De Rham cohomology; why "co"-homology
- Intuitive meaning of singular homology: use cycles to detect the topology; `same' if coboundary
- Stokes theorem from perspective of duality between forms and cycles
- Intuitive reason for defining de Rham cohomology use $\int_c \omega$ detects non-trivial cycles. $\int_{C_1} \omega = \int_{C_2} \omega$ if ω closed & $C_1 \sim C_2$.
- Cycles: easy to imagine, difficult to write down. $\int_c d\eta = 0.$
- Example: punctured three-space and n-space
- Cup product is well-defined (Prob. 1) => algebra
- Pull back; functoriality (category of manifolds -> category of vector spaces)
- Degree zero cohomology
- Hodge theorem (Prob. 17.2)

$$I = \Delta G + \mathrm{Proj}_{\ker \Delta}.$$





$$(r\sin\varphi\cos\theta,\ r\sin\varphi\sin\theta,\ r\cos\varphi)$$

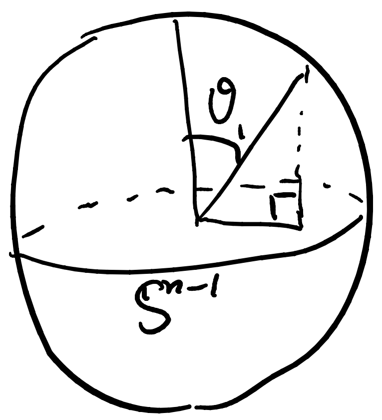
$$dx = \sin\varphi\cos\theta\, dr + r\cos\varphi\cos\theta\, d\varphi - r\sin\varphi\sin\theta\, d\theta.$$

$$dy = \sin\varphi\sin\theta\, dr + r\cos\varphi\sin\theta\, d\varphi + r\sin\varphi\cos\theta\, d\theta$$

$$dz = \cos\varphi\, dr - r\sin\varphi\, d\varphi.$$

$$d\mathrm{vol}_{S^2} = \sin\varphi\, d\varphi \wedge d\theta$$

$$d\mathrm{vol}_{S^n} = (d\theta_1) \wedge (\sin\theta_1)(d\theta_2) \wedge (\sin\theta_1 \sin\theta_2)\, d\theta_3 \wedge \dots \wedge (\sin\theta_1 \dots \sin\theta_{n-1})\, d\theta_n$$



S^n