- Homotopy of maps
- Homotopy of chain maps
- Homotopic chain maps induce the same map on cohomology
- From homotopy of maps to that of chain maps
- Homotopic maps induce the same pullback on cohomology
- Homotopic manifolds have same cohomology (eg circle and punctured plane)
- Contractible manifolds have trivial higher cohomology: Poincare lemma
- Eg: star shaped (Prob. 17.4)
- H^1 = Hom(first fundamental group, R)

homotopy of chain maps

$$\begin{array}{ccccccc} & & \Omega^\cdot(N) & & & & \\ & & \| & & & & \\ \cdots & & A^\cdot & \xrightarrow{d} & A^{\cdot+1} & & \cdots \\ & & \downarrow & \swarrow h & \downarrow f, g & & \\ \cdots & & B^\cdot & \xrightarrow{d} & B^{\cdot+1} & & \cdots \\ & & \| & & & & \\ & & \Omega^\cdot(M) & & & & \end{array}$$

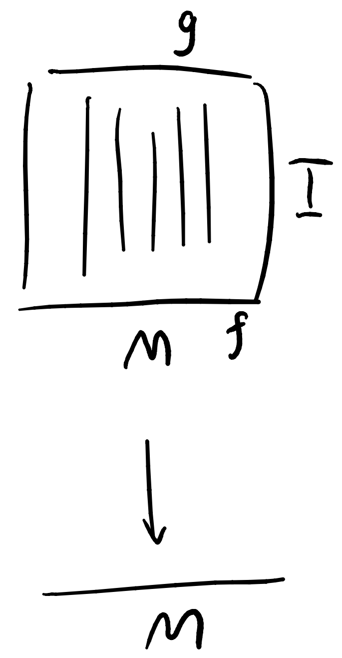
$$g - f = dh + hd.$$

homotopy of maps

$$M \times I \xrightarrow[H]{} N$$

$$\int_I \cdot$$

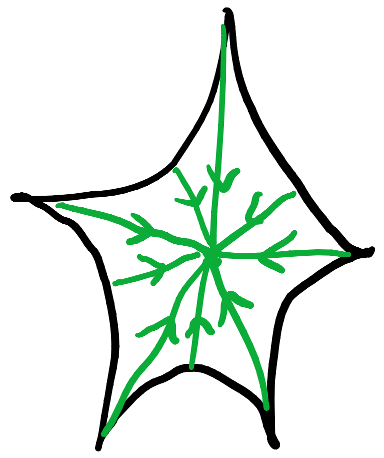
$$\Omega^\cdot(N) \xrightarrow[H^*]{} \Omega^\cdot(M\times I) \longrightarrow \Omega^{\cdot-1}(M), \qquad h : \Omega^\cdot(N) \to \Omega^{\cdot-1}(M)$$



$$d \int_I \underbrace{H^* \omega}_{dt\wedge(\dots)+\underbrace{(\dots)}_{\text{no } dt}} = \int_I H^*(d\omega) - (g-f).$$

star shaped:

$$H(x,t) = tx.$$



$$\eta = \sum_I{}' \sum_{q=1}^{p} (-1)^{q-1} \left(\int_0^1 t^{p-1} \omega_I(tx)\, dt \right) x^{i_q}\, dx^{i_1} \wedge \cdots \wedge \widehat{dx^{i_q}} \wedge \cdots \wedge dx^{i_p}$$

$$d\eta = \omega.$$

$$d_M \int_I \underbrace{(dt\wedge a(t) + b(t))}_{\varphi} = \int_I dt \wedge d_M a$$

$$= -\int_I d(dt \wedge a)$$

no dt

$$\cancel{d_M b} + dt\wedge \partial_t b$$

$$= -\int_I d(dt\wedge a + b) + \int_I db$$

$$= -\int_I d\varphi + \underbrace{(b_1 - b_0)}_{i_1^*\varphi - i_0^*\varphi}$$