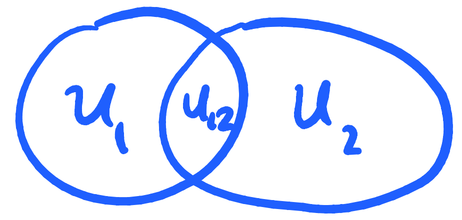
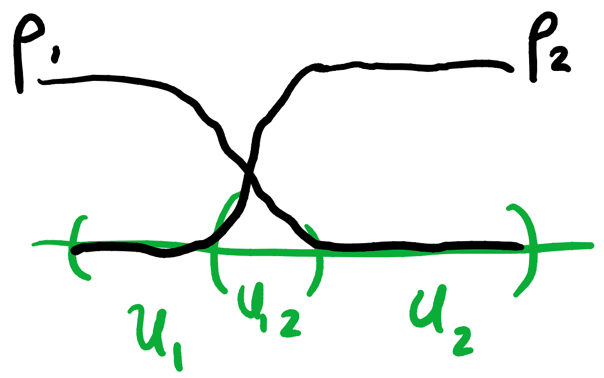
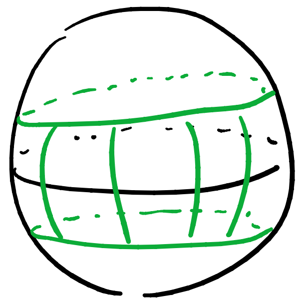
- The short exact sequence of de Rham complexes $0 \to \Omega^\bullet(M) \xrightarrow{(i_1^*,\, i_2^*)} \Omega^\bullet(U_1) \oplus \Omega^\bullet(U_2) \xrightarrow{r_1^* - r_2^*} \Omega(U_{12}) \to 0.$
- The Mayer-Vietoris sequence
- Partition of unity to prove the short exact sequence (last map is onto)
- Connecting homomorphism: from short exact sequence to long exact sequence
- Example: sphere. Write down an explicit generator. $\mathrm{vol}_{S^{n-1}}$.
- Alternating sum of all dimensions in long exact sequence = 0. Also can cut into pieces at 0.
- Manifolds of different dimensions cannot be homeomorphic. (Consider contractible open set - {p})
- Prob. 17.5, 6, 7





$$r_1^*(\underbrace{\rho_2\omega_{12}}_{\text{on } U_1}) + r_2^*(\underbrace{\rho_1\omega_{12}}_{\text{on } U_2}) = \omega_{12}.$$

$$
\begin{array}{l}
0 \to \overset{\mathbb{R}}{H^0(S)} \to \overset{\mathbb{R}}{H^0(U_1)} \oplus \overset{\mathbb{R}}{H^0(U_2)} \to \overset{\mathbb{R}}{H^0(U_{12})} \\
\to H^1(S) \to \overset{0}{H^1(U_1)} \oplus \overset{0}{H^1(U_2)} \to \overset{0}{H^1(U_{12})} \\
\Rightarrow 0 \qquad \vdots \qquad \vdots \\
\Rightarrow 0 \\
\to H^{n-1}(S) \to \overset{0}{H^{n-1}(U_1)} \oplus \overset{0}{H^{n-1}(U_2)} \to \overset{\mathbb{R}}{H^{n-1}(U_{12})} \\
\xrightarrow{\simeq} H^n(S) \to \overset{0}{H^n(U_1)} \oplus \overset{0}{H^n(U_2)} \\
\Rightarrow \mathbb{R}
\end{array}
$$



$U_1 \cap U_2$

$U_{12} \sim S^{n-1}$.

$$0 \to A_1 \to A_2 \to \dots \to A_k = 0.$$

$$\dim_k = \ker_k + \mathrm{im}_{k+1}.$$

$$\overset{}{\underset{\text{exact}}{=}} \ker_k + \ker_{k+1}.$$

$$\sum_k (-1)^k \dim_k = 0.$$