Sunday, December 27, 2015 8:26 PM

- Compactly supported forms and cohomology algebra (important for Poincare duality for non-compact manifolds)
- Eg. H^n(R^n) = 0. But it should not if there is duality!
- Note: pull-back only well-defined for proper maps (so Poincare lemma is non-trivial)
- Have push-forward for fiber bundles (which commutes with d)
- Compactly supported Poincare lemma
- R^n. (Integration)
- Homotopic proper maps induce same pull-back on cohomology (NOTE: NEED HOMOTOPY TO BE PROPER)
- Top compactly supported cohomology is one-dimensional. (by integration which is only defined for oriented manifolds)
- Non-orientable case: top cohomology = 0. (Thus don't have Poincare duality.)
(Pull back to oriented double cover, which is injective on cohomology)

$$\boxed{d_M \int_F \omega = (-1)^{\dim F} \int_F d_E \omega.} \qquad \begin{matrix} F \to E \\ \downarrow \\ M \end{matrix}$$

$$\int_{\mathbb{R}^n} \underbrace{\omega}_{\text{closed}} = 0 \Rightarrow \omega = d\eta.$$

Key: make η cpt. supp.

$d\eta = 0$ far away. If $\deg \eta < n-1$, $H^{\deg \eta}(\mathbb{R}^n - pt) = 0$

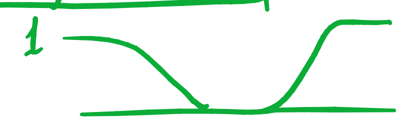
$\Rightarrow \eta = d(\text{stuff})$.

If $\deg \eta = n-1$, $\int_{S^{n-1}} \eta = \int_B d\eta$

$= 0$

$\Rightarrow \eta = d(\text{stuff})$.

Then take $\boxed{\eta - d(\rho \cdot \text{stuff})}$.



$$\Omega_c^\cdot(M) \xrightarrow{H^*} \Omega_c^\cdot(M \times I) \xrightarrow{\int_I} \Omega_c^\cdot(M).$$

$\int_M : H_c^{top} \to \mathbb{R}$ is inj:

$\int_M \underbrace{\omega}_{\text{cptly supp top form}} = 0 \Rightarrow \omega = d\underbrace{\eta}_{\text{cptly supp}}.$

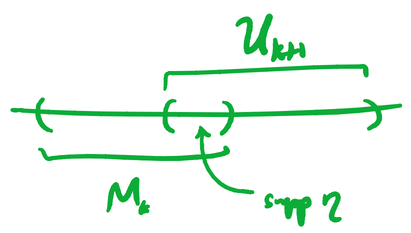
Induction on |chart|.

For $\text{supp}(\omega) \subset \underbrace{U_1 \cup \dots \cup U_k}_{M_k} \cup U_{k+1}$.

part. of unity $\varphi + \psi = 1$.

$\int_{M_k} \varphi\omega = c$. Want 0 to do induction.

η : bump form in $M_k \cap U_{k+1}$ st. $\int_{\text{supp}} \eta = 1$



$\int_{M_k} (\varphi\omega - c\eta) = 0 \Rightarrow \varphi\omega - c\eta = d(\underbrace{\text{stuff}}_{\text{supp in } M_k})$.

$\int_{U_{k+1}} \underbrace{\psi\omega + c\eta}_{\text{supp in } U_{k+1}} = \int_M \omega - (\varphi\omega - c\eta) = 0 \Rightarrow \psi\omega + c\eta = d(\underbrace{\text{stuff}}_{\text{supp in } U_{k+1}})$.

$\omega = d(\text{stuff}_1 + \text{stuff}_2)$. ∗

$\pi : \tilde{M} \to M$.

π^* injective:

If $\pi^*\omega = d\tilde{\alpha}$,

then $\omega = \frac{1}{2} d(\underbrace{\tilde{\alpha} + \sigma^*\tilde{\alpha}}_{\text{inv. form}})$.

$Im(\pi^*) = 0$:

$\underbrace{\sigma}_{\text{ori. reversing}}^*(\pi^*\omega) = \pi^*\omega$

$\Rightarrow \int_{\tilde{M}} \pi^*\omega = 0.$