- Definition by pull-back
- The same as (signed) number of points in the preimage of a regular value => integer (use bump volume form)
- Degree under composition; homotopic invariance
- Eg. Polynomial: P^1 to P^1 (Prob. 17.10) => why it is called degree
- Brower fixed-point theorem
- Proper map version (Prob. 17.11)
- Prob. 17.12, 13.

$$\mathbb{R} \overset{\int_N}{\simeq} H^n(N) \xrightarrow{f^*} H^n(M) \overset{\int_M}{\simeq} \mathbb{R}$$

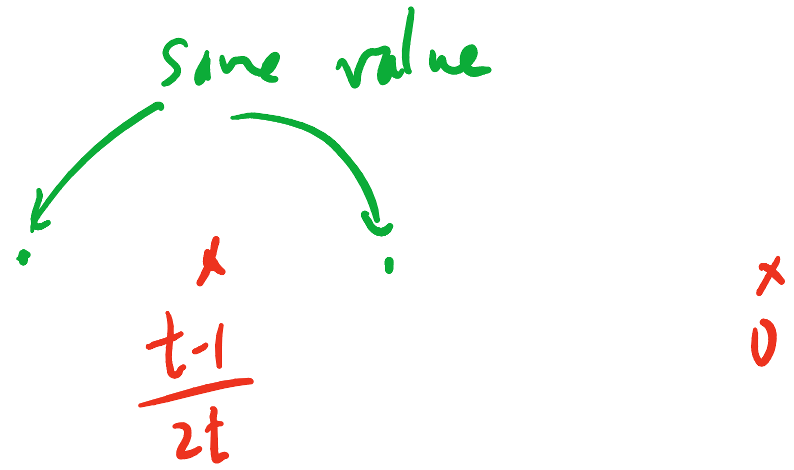
homo : deg.

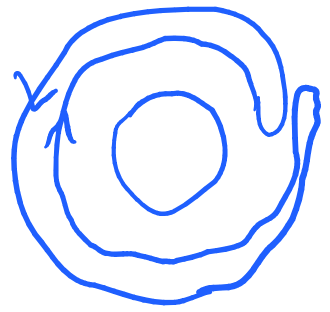
Homotopy needs proper!

e.g. $z \mapsto z^2 : \mathbb{C} \circlearrowleft$

$$H = tz^2 + (1-t)z = t\left(\left(z + \frac{1-t}{2t}\right)^2 - \left(\frac{1-t}{2t}\right)^2\right)$$

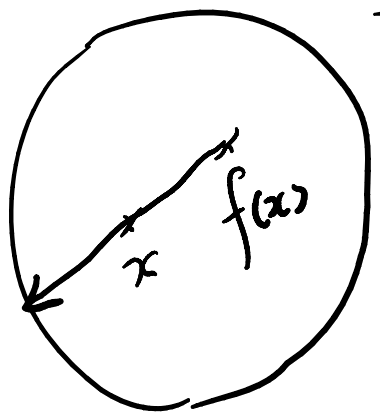
$$H^{-1}\{0\} = \{(z,t) \in \mathbb{C}\times[0,1] : z = 1 - \frac{1}{t}\} \text{ non-cpt.}$$



e.g.  $S^1 \circlearrowleft$ onto, $\deg = 0$

Brower fixed-point theorem:

$\bar{B}^n \overset{f}{\circlearrowleft}$ has fixed pt.



$$\tilde{f}(x) \triangleq \frac{x - f(x)}{\|\cdot\|} : \bar{B} \to \partial B$$

with $\tilde{f}|_{\partial B} \sim Id_{\partial B}$

$$\frac{x - tf(x)}{\|\cdot\|}$$

$$\begin{array}{ccc} M^{n+1} & \xrightarrow{\tilde{f}} & N^n \\ \cup & \nearrow & \\ \partial M & & \end{array}$$

$$\int_{\partial M} \tilde{f}^* dvol_N = \int_M d(\tilde{f}^* vol_N) = 0. \quad \therefore \deg \tilde{f}|_{\partial B} = 0.$$

$\deg = 1$

contradiction!

$tz^2+(1-t)z$ does not define a C^1 homotopy $\mathbb{P}^1\times[0,1]\longrightarrow\mathbb{P}^1$.

Around $(z=\infty, t=1)\longmapsto \infty\in\mathbb{P}^1$:

$$\frac{w^2}{t+(1-t)w} \quad \text{around } w=0 \ \& \ t=0$$

$$\|$$

$$\frac{w^2}{-tw+t+tw}\sim\frac{w^2}{t+w}$$

$$d(\cdot\cdot)=\frac{2(t+w)w\,dw-w^2(dt+dw)}{(t+w)^2}$$

$$=\frac{(2tw+w^2)dw}{(t+w)^2}-\frac{w^2\,dt}{(t+w)^2}$$

$$\frac{1}{\left(\frac{t}{w}+1\right)^2}$$

NOT cont. at $t=w=0$.