Motivation: a more intuitive way to define homology. Poincaré dual.
- The singular chain complex (simplex, chain, face, boundary map, $d^2 = 0$)
- Eg. Circle in $\mathbb{R}^2 - 0$ as a singular cycle
- Push forward and functoriality ($f: M \to N$ commutes with $\partial$)
- Homotopy
- The cochains and coboundaries $C^* = H_{co} (C, \mathbb{R})$
- Mayer-Vietoris sequences
- Can replace continuous by smooth chains (Whitney approximation $\Rightarrow$ smooth map homotopic to each continuous map; get chain map)

\[
0 \to C_* (U \cap V) \to C_* (U) \oplus C_* (V) \to C_* (M) \to 0
\]

\[
0 \to C^* (M) \to C^* (U) \oplus C^* (V) \to C^* (M \cup V) \to 0
\]

From homotopy to chain homotopy
\[
H: M \times I \to N
\]

\[
\tilde{h}(\Delta) = (-1)^n \Delta \times I.
\]

Subdivide $\Delta \times I$ into simplices.