De Rham theorem and Poincare dual

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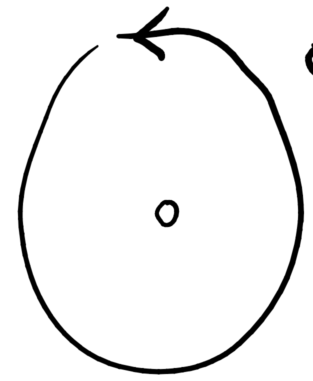
- Motivation: "particle-field duality"
- The de Rham theorem
- Example of Poincare dual
- The map is well-defined and functorial
- Proof: the same as PD.
- Calibrated submanifolds (Prob. 18.4) -> absolutely minimal submanifold in the same class. (Much easier than mean curvature flow)
- Eg. Inner circle of a torus in R^3 (Prob. 18.5)
- Prob. 18.1, 2.

$$\boxed{H_\cdot(M,\mathbb{R}) \xrightarrow[\sim]{\int_\cdot \,\cdot} \left(H^\cdot(M,\mathbb{R})\right)^* \underset{P.D.}{\overset{\int_M (\cdot)\wedge(\cdot)}{\simeq}} H_c^{n-\cdot}(M,\mathbb{R})}$$

e.g.

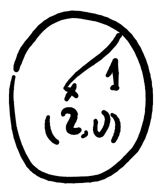


$S^1 \subset \mathbb{R}^2 - \{0\}$.

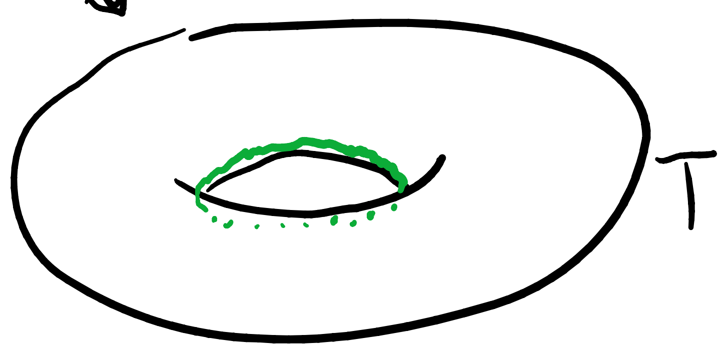
$[S^1] = \rho(r)\, d\theta$

s.t. $\int_0^\infty \rho(r)\, r\, dr = \frac{1}{2\pi}$.

Revolving



about z-axis



cal. form: $d\theta|_T$.

(the scale is important)

unit vector: $a\frac{\partial}{\partial r} + \frac{b}{r}\frac{\partial}{\partial\theta}$, $a^2+b^2 \le 1$.

$d\theta(\cdot\cdot) = \frac{b}{r} \le \frac{1}{r} \le 1$ on T.

Note: $d\theta$ on $\mathbb{R}^3 - (z\text{-axis})$ is not a cal. form.

Calibration: p-form ω w/ $\begin{cases} d\omega = 0 \\ \omega_x(v_1 \ldots v_p) \le 1 \quad \forall \text{ orthonormal } p\text{-frame } \{v_1 \ldots v_p\}. \end{cases}$

$S \subset M$ calibrated: $\omega|_S = d\mathrm{vol}_S$.

$\Rightarrow \forall S' \in [S]$,

$$\mathrm{Vol}_S = \int_S d\mathrm{vol}_S = \int_S \omega = \int_{S'} \omega \le \int_{S'} d\mathrm{vol}_{S'} = \mathrm{Vol}_{S'}.$$

e.g. (M, J, ω, g) Kähler. ($d\omega = 0$, $g(u,v) = \omega(u, Jv)$, $g(Ju, Jv) = g(u,v)$.)

Then ω is a calibration form.

Calibrated submflds: complex submflds.