De Rham theorem and Poincare dual

- Motivation: "particle-field duality"
- The de Rham theorem
- Example of Poincare dual
- The map is well-defined and functorial
- Proof: the same as PD.
- Calibrated submanifolds (Prob. 18.4) -> absolutely minimal submanifold in the same class. (Much easier than mean curvature flow)
- Eg. Inner circle of a torus in $\mathbb{R}^3$ (Prob. 18.5)
- Prob. 18.1, 2.

**De Rham theorem and Poincare dual**

$$H_1(M, \mathbb{R}) \cong \left( H^1(M, \mathbb{R}) \right)^* \cong H^{n-1}_c(M, \mathbb{R})_{PD}$$

**Example of Poincare dual**

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**Proof:** the same as PD.

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Prob. 18.1, 2.

Calibration: $p$-form $\omega$ with $\{dw=0$, $w_x(v_1...v_p) \leq 1\}$ orthonormal frame $\{v_1...v_p\}$.

$S \subset M$ calibrated: $\omega|_S = d\nu|_S$.

$V\nu|_S = \int_S d\nu|_S = \int_S \omega = \int_{S'} \omega = \int_{S'} d\nu|_{S'} = V\nu|_{S'}$.

Eg. $(M, J, \omega, g)$ Kähler: $(dw=0$, $g(u,v) = \omega(u,Jv)$, $g(Ju,Jv) = g(u,v).$)

Then $\omega$ is a calibration form.

Calibrated submanifolds: complex submanifolds.