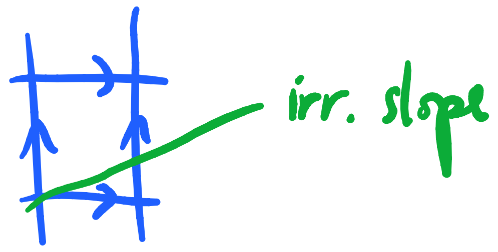
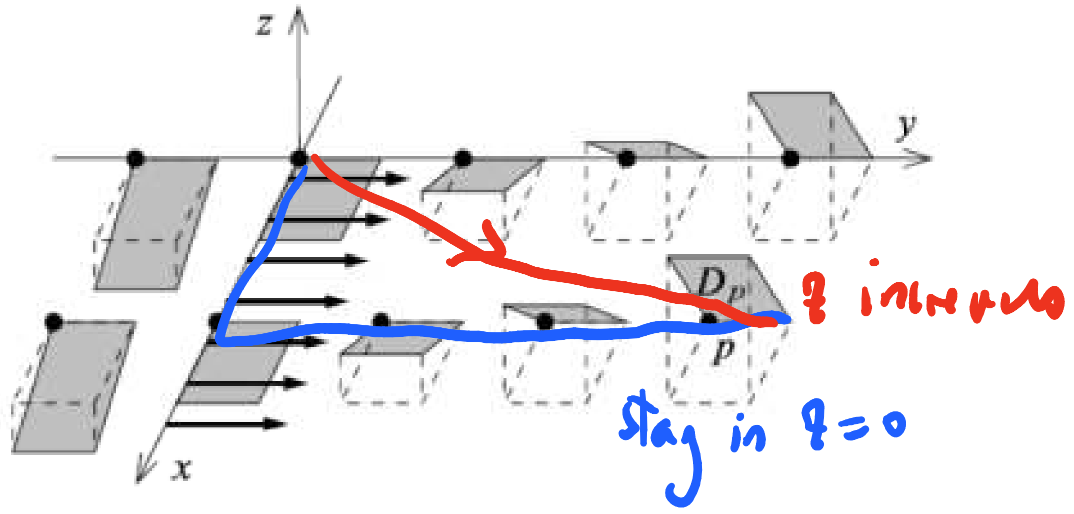
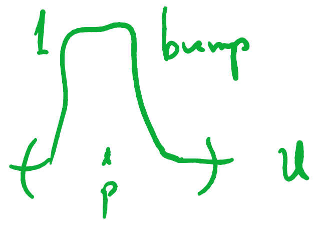
- Distribution D = subbundle of TM
- (local) integral manifold N: TN = D (Don't need embedded globally) e.g.

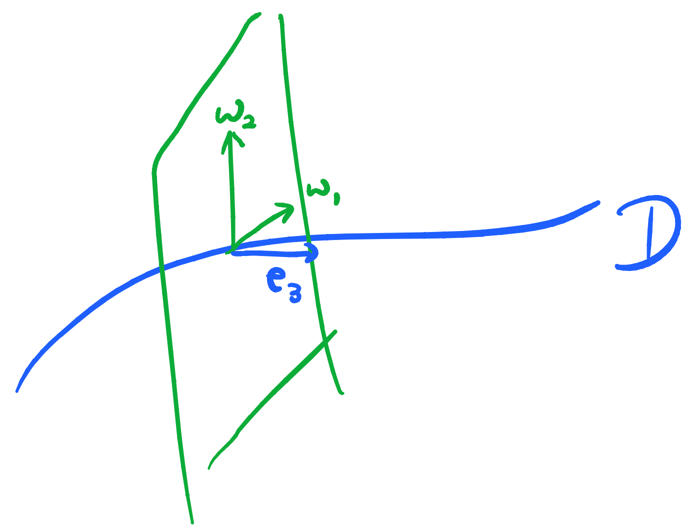




- Examples
    - Vector field and its integral curve
    - Sphere: $\langle \sum x_i \frac{\partial}{\partial x_i} \rangle^{\perp}$
    - Not integrable: $X = \partial_x + y\partial_z$, $Y = \partial_y$
- Necessary: involutive $[X,Y] \in \Gamma_{loc}(D)$
- Involutive <=> $\Gamma(D)$ is Lie subalgebra $\Leftrightarrow$ use

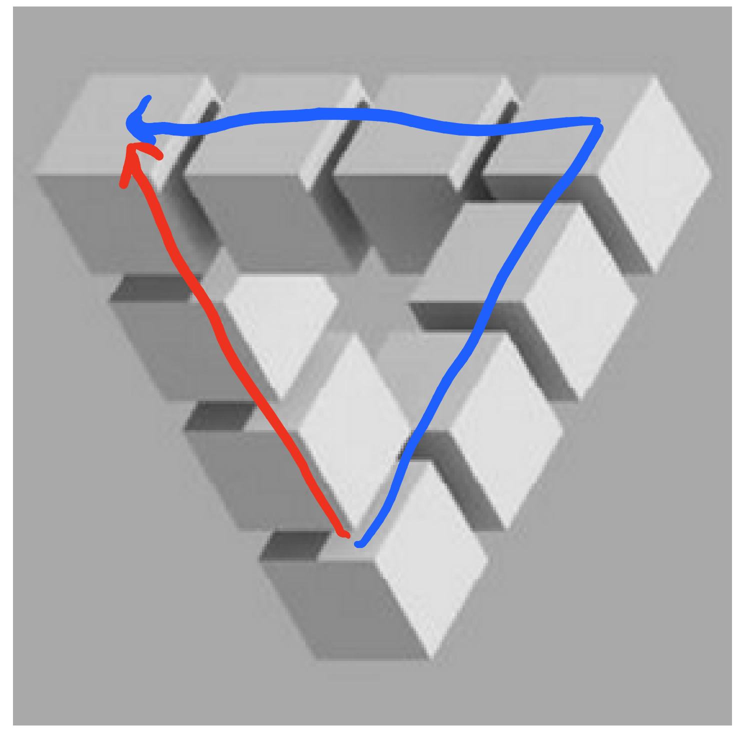


- integrable => involutive
- In terms of local frames $[e_i, e_j] \in \Gamma_{loc}(D)$
- D defined by local defining one-forms: $\ker\omega_1 \cap \dots \cap \ker\omega_{n-k}$ (use loc. frame)
- Analog: submanifold S locally defined by functions $f_1 \dots f_k$ equivalently the ideal $C^\infty(M)\cdot\{f_1, \dots, f_k\} = \{f|_S = 0\}$



- P-form annihilates D <=> $\sum_i \omega_i \wedge \beta_i$ (use loc. frame)
  Hence {ann. forms} = $\Omega(M)\cdot\{\omega_1, \dots, \omega_{n-k}\}$ (locally) (ideal)
  Integrability: want $\omega_i = df_i$
- Involutive <=> {ann. forms} is closed under d (diff. ideal) $(d\eta)(X,Y) = X(\eta(Y)) - Y(\eta(X)) - \eta([X,Y])$.
- Need to talk about sheaf in the holomorphic setting
- Prob. 19.2

Prob. 19.3: ω nowhere-0 1-form.
$\exists$ loc. int. factor μ s.t. $\mu\omega = df \Leftrightarrow d\omega \wedge \omega = 0$.



Motivation: vector field always integrates to give a local flow
$\longrightarrow$ 1 para. action $\exp tX$.
Generalize this to $\mathbb{R}^n \curvearrowright M$?
Need integrability condition!
Translational symmetry.
conserved quantity: momentum.