• **Frobenius theorem**: involutive $\iff$ integrable. $\leq$ is trivial.
• Already known: $V_i, \ldots, V_k$ form a basis of $\mathbb{R}^k$ s.t. $V_i = \partial_i$. ($p^iv = p^jv$ as $[V_i, V_j] = 0$)
• Key: $D$ closed under $[\ ]$ => can choose local frame $\{V_i\}$ such that $[V_i, V_j] = 0$

Take local coord. $x_i$, loc. frame $D = \langle x_1, x_n \rangle$ s.t. $D \ni \partial_i, \ldots, \partial_n$.

$$\pi : U \to \mathbb{R}^k$$

Define $V_i$ by $V_i \mapsto \partial_i$.

$$d\pi ([V_i, V_j]) = [\partial_i, \partial_j] = 0 \Rightarrow [V_i, V_j] = 0.$$  

• Integral manifold $S$ is weakly embedded:
  any smooth $N \to M$ with image in $S$ is a smooth map $N \to S$

• Integrable distribution $\iff$ foliation: collection of disjoint connected immersed submanifolds whose union is the whole space, and can take local coordinates such that defined by $x_{(k+1)} = \ldots = x_n = 0$.

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