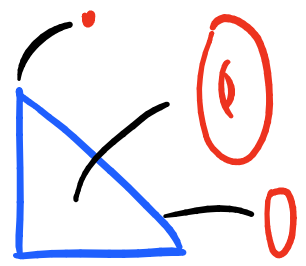
First class overview

Friday, January 15, 2016 8:48 PM

Toric geometry and mirror symmetry

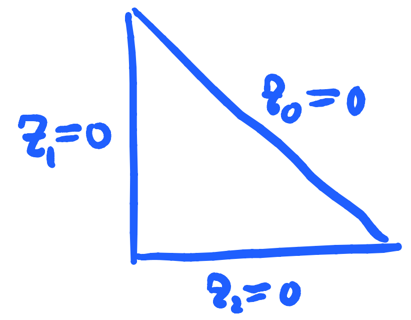
- Motivation: `see' high-dimensional (Kaehler) geometry from combinatorics of polytopes.
- A good starting point to learn Kaehler and algebraic geometry
- Can do a lot of experiments!

e.g. $\mathbb{CP}^n \longleftrightarrow$ 

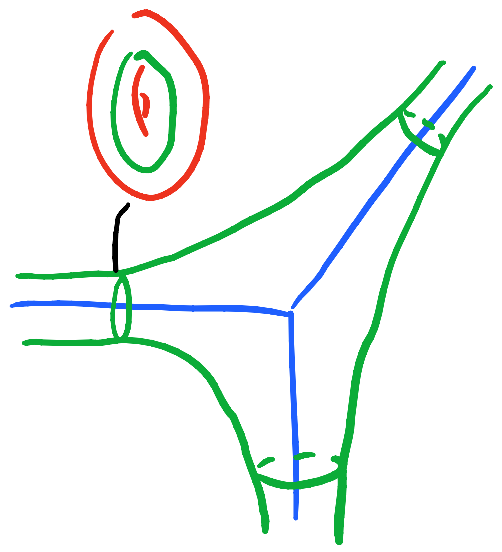
$= (\mathbb{C}^{n+1}-\{0\})/\mathbb{C}^\times$

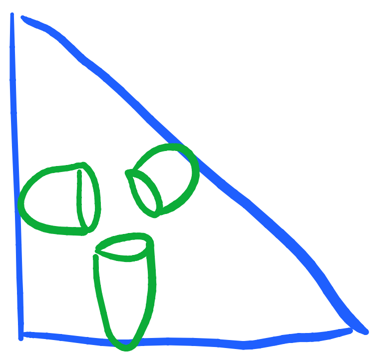
$= \mathrm{Proj}\,\mathbb{C}[z_0,\dots,z_n]$

- torus fib. (torus action, torus fibers shrink)
- divisors & lin. equiv. (toric)



- Int$^{\underline{sn}}$ of (toric) cycles.
- Sections of $\mathcal{O}(k) \longleftrightarrow$ lattice points of Δ.
- Holomorphic curves $(\Sigma \to X) \longleftrightarrow$ tropical curves in $\mathbb{R}^2 \subset \Delta$
- Holomorphic discs $(\text{disc} \to (X,T)) \longleftrightarrow$ tropical discs





Tropical geometry is originated from computer science!

GW, SYZ, mirror symmetry

$X^n \supset Y$ hyperplane section $\Rightarrow H^k(X) \to H^k(Y)$ iso. $k<n-1$, inj. $k=n-1$.

- More complicated geometry can be studied by Leftschetz hyperplane theorem, toric degeneration, or Okounkov body.
- There will be many examples. We will illustrate using one example, and the concepts and methods easily extend to general case.
- We can have a sense of many classical and modern aspects of algebraic geometry.
- We will skip many proofs, but instead giving the ideas of proof or a way of convincing ourselves.
- There will be a simple assignment after each class; you should turn in on the next Tuesday. NO FINAL EXAM. You may be asked to give a presentation on an example in the last week.