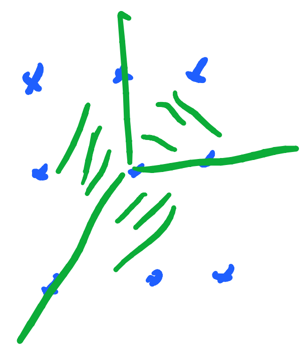
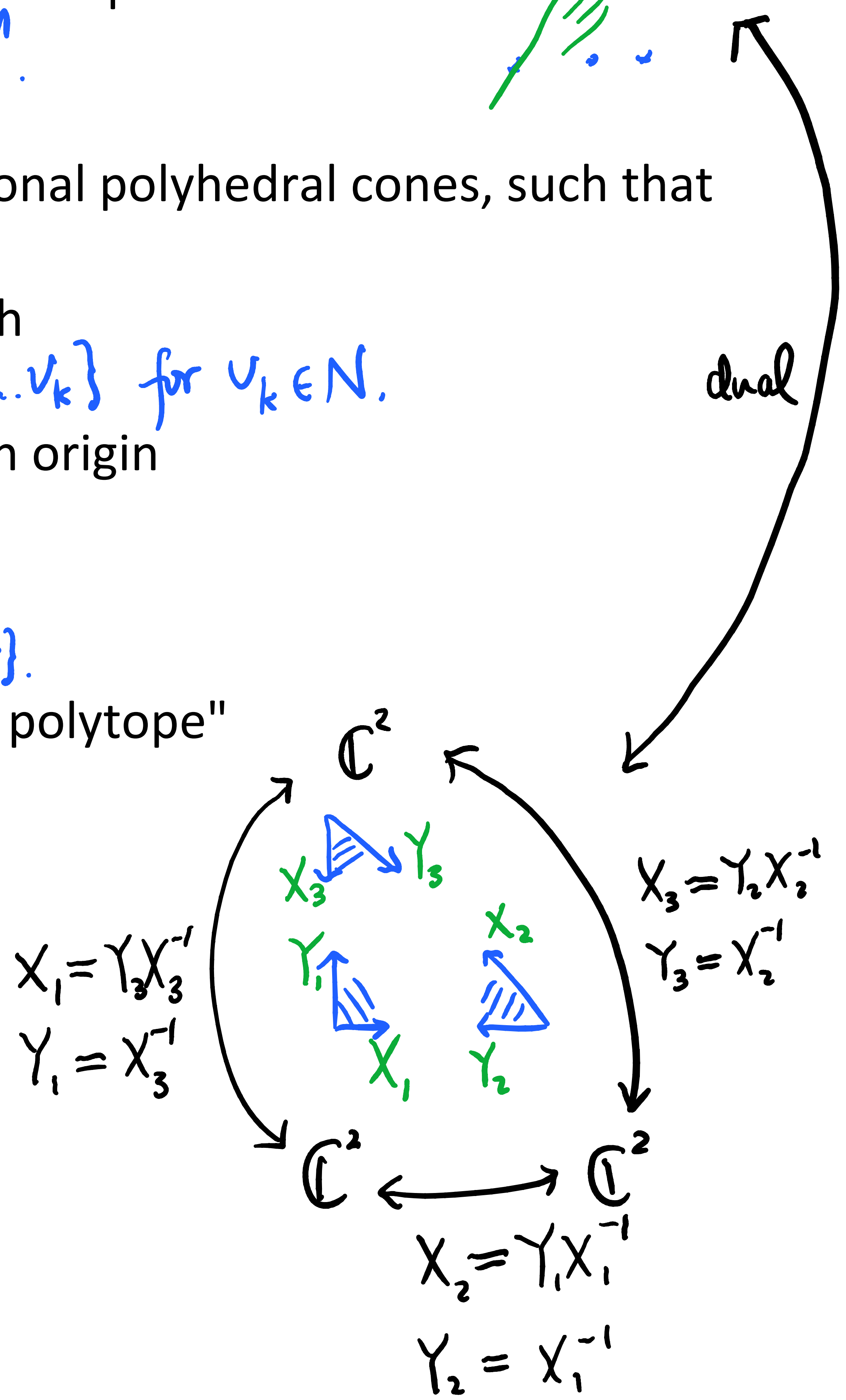
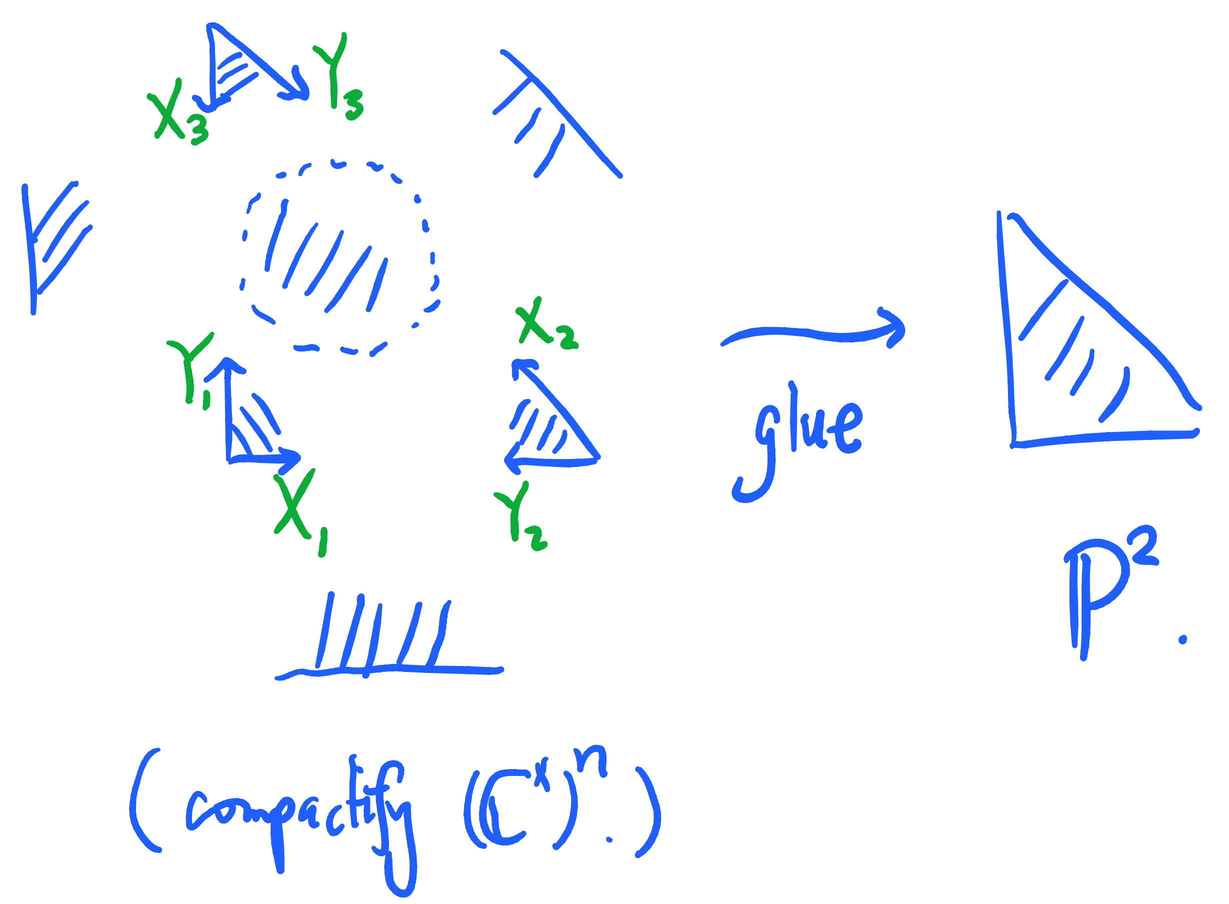
Monday, January 11, 2016 3:02 PM



- Main statement: fan -> variety. Useful to keep P^2 in mind
- Lattice $N \simeq \mathbb{Z}^n$. $N_{\mathbb{R}} = N \otimes \mathbb{R} \simeq \mathbb{R}^n$.
  We want basis-free
- Fan Σ: collection of strongly convex rational polyhedral cones, such that
  - face of a cone in Σ is still in Σ
  - intersection of cones in Σ is face of each
- Rational polyhedral cone: $\sigma = \mathbb{R}_{\geq 0}\{v_1 \dots v_k\}$ for $v_k \in N$.
  Strongly convex: contains no line through origin
  Face: $\mathbb{R}_{\geq 0} S$ for $S \subset \{v_1 \dots v_k\}$.
- Dual lattice $M = \mathrm{Hom}(N, \mathbb{Z}) \simeq \mathbb{Z}^n$.
- Dual cone $\sigma^\vee = \{ \nu \in M_{\mathbb{R}} : \langle v, \nu \rangle \geq 0 \ \forall v \in \sigma \}$.
- Dual cones patch together to form "dual polytope"
- Simplicial cone $k = n$, $\{v_1 \dots v_k\}$ l.i.
- Standard cone $\mathbb{Z}\{v_1 \dots v_n\} \simeq N$.
- C^n -> standard cone
- How to glue: P^2
- P^1 $\longleftarrow \bullet \longrightarrow$ $\overset{\text{dual}}{\longleftrightarrow}$ $[\longmapsto \ \longleftarrow]$
- Affine piece $\sigma \longrightarrow \mathrm{Spec}(\mathbb{C}[\sigma^\vee \cap M]) \triangleq U_\sigma$.
- Ex $\{0\} \to (\mathbb{C}^\times)^n$, $\{(1, 0 \dots 0)\} \longrightarrow \mathbb{C} \times (\mathbb{C}^\times)^{n-1}$
- $\tau \subset \sigma \longrightarrow U_\tau \subset U_\sigma$.
- Ex: P1*P1, P3



$\mathbb{P}^2 \ni [x : y : z]$
$(X_1, Y_1) = \left(\frac{x}{z}, \frac{y}{z}\right)$
$(X_2, Y_2) = \left(\frac{y}{x}, \frac{z}{x}\right)$
$(X_3, Y_3) = \left(\frac{x}{y}, \frac{z}{y}\right)$.



Standard reference: Fulton - Introduction to toric varieties