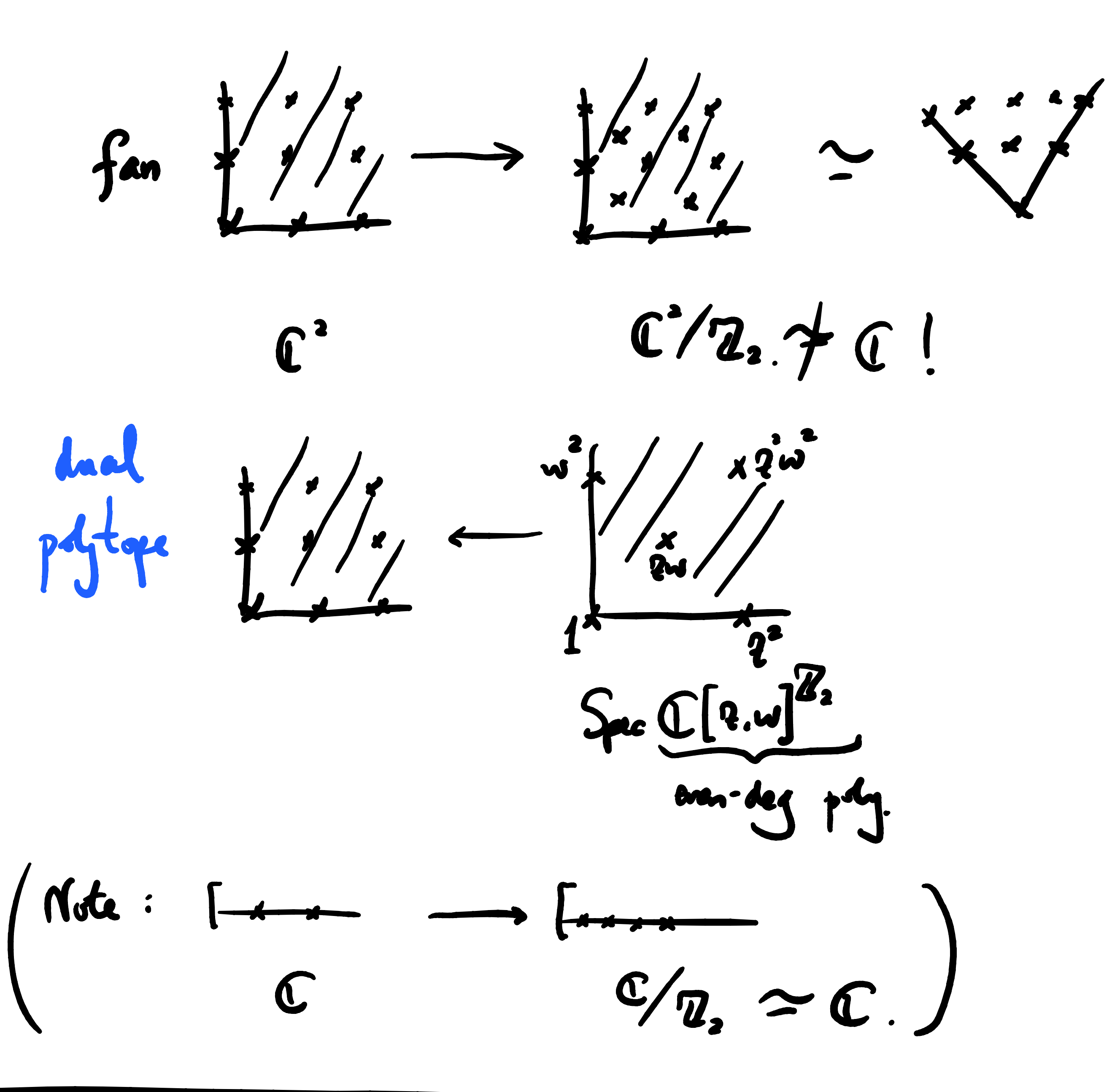
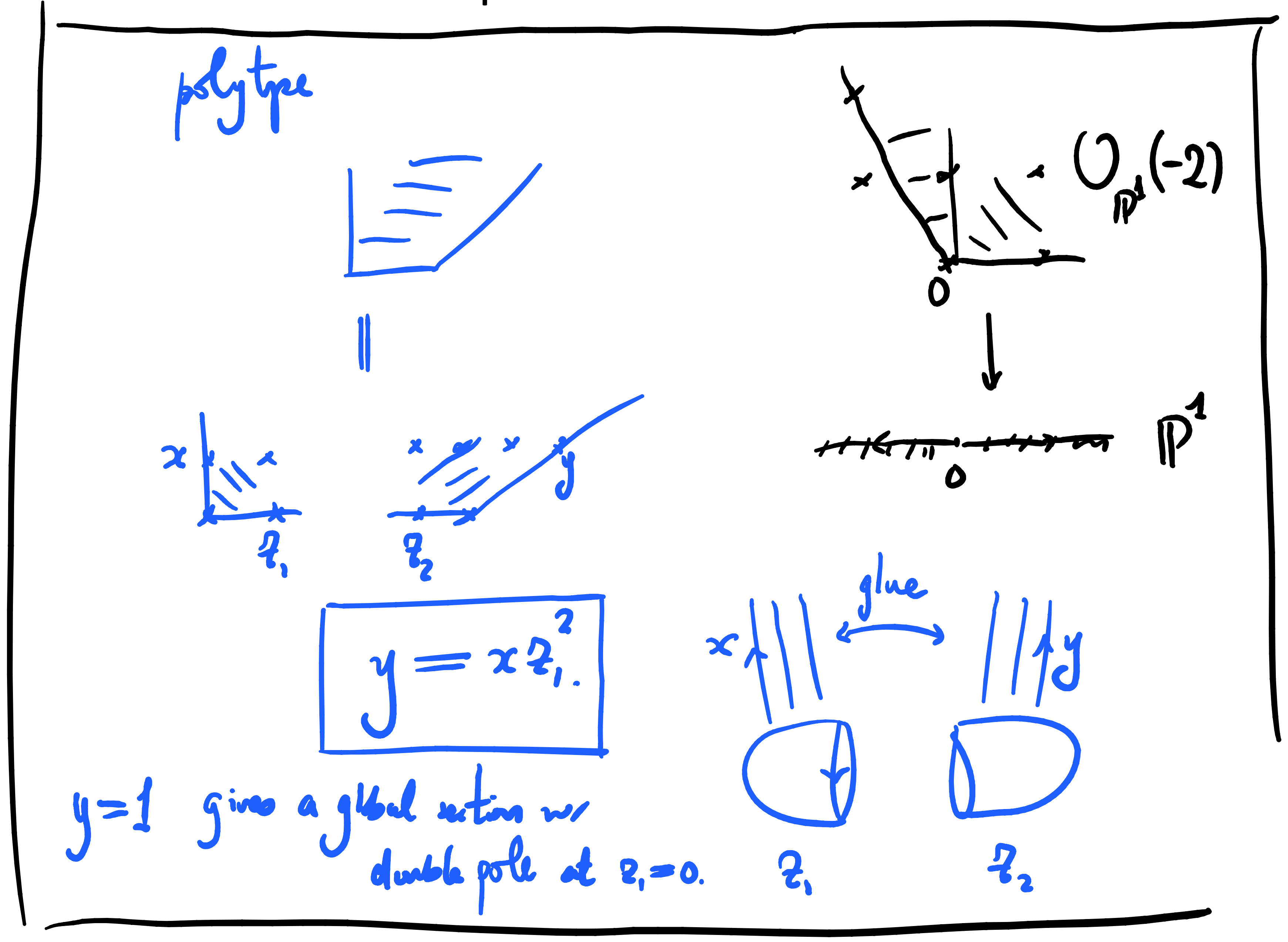
Toric morphisms

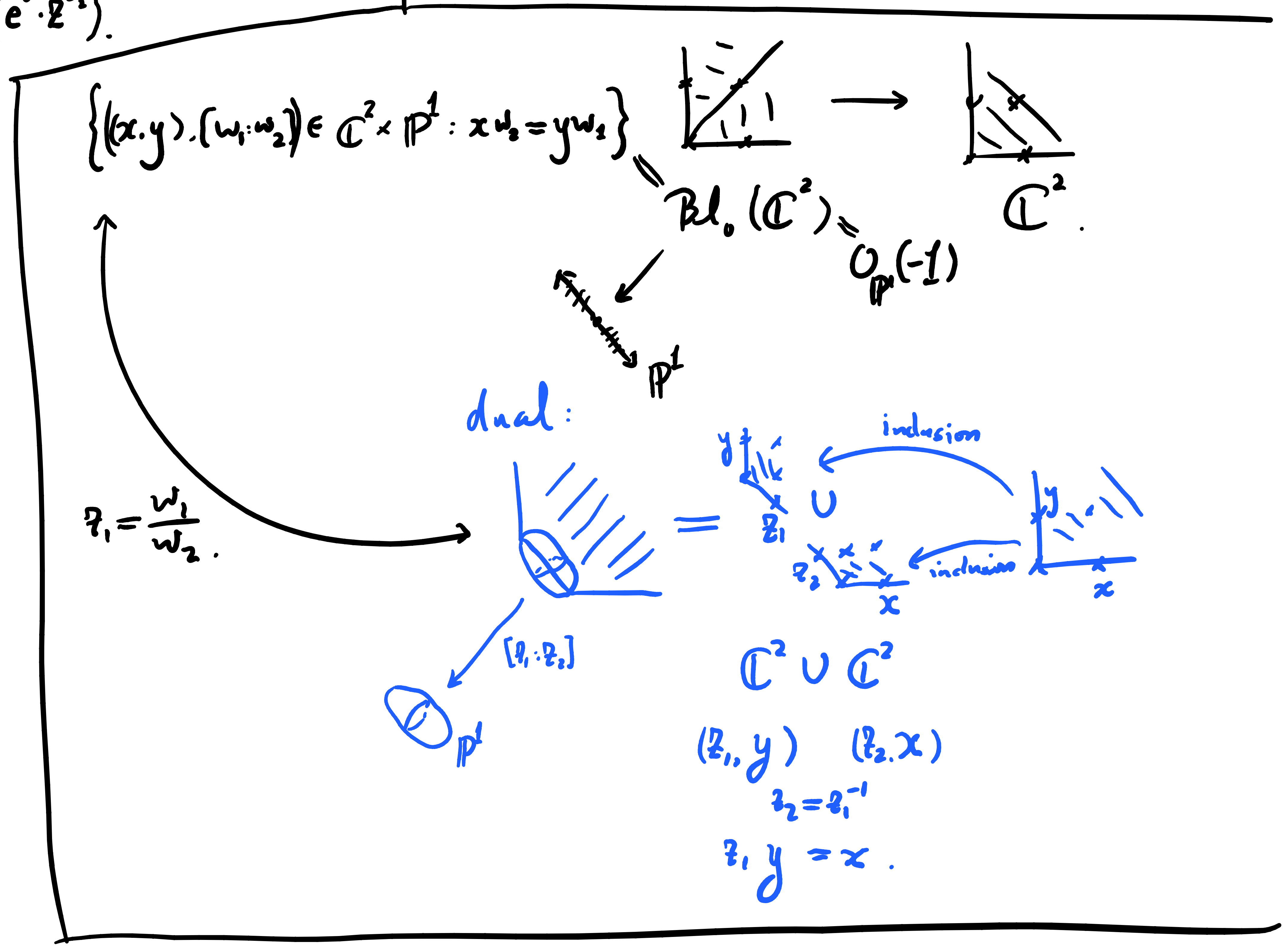
Monday, January 11, 2016 3:28 PM

Motivation: give useful constr. in A.G.

- Recall P^2. Fan <-> polytope. The fan indicates how to compactify $(\mathbb{C}^\times)^n$
- Fan morphisms $(N_1,\Sigma_1)\xrightarrow{f}(N_2,\Sigma_2)$, $f(\sigma)\subset\tau\in\Sigma_2$, $\longrightarrow \mathbb{C}[\sigma^\vee\cap M_1]\leftarrow\mathbb{C}[\tau\cap M_2]$.
- Refine the lattice: C^2 -> C^2/Z2
- Blow-up of C^2. Blow-up at a fixed point <-> "add a ray"
- O(k) bundle over P^1
- A_n, A_\infty
- Canonical line bundle K_{X}
- Toric compactification F_k of O(k): P^1 bundle
- Bundle map from K_X to X
- O(-1,-1)->conifold, also -> P^1, flop
- Compact toric <-> complete fan (every direction is compactified)
- Proper morphism <-> $\varphi^{-1}(|\Sigma|)=|\Sigma'|$.
- Torus action $v\in N_{\mathbb{C}}/N$, $e^v\cdot z^\nu=\exp(2\pi i\,(v,\nu))\cdot z^\nu$. $e^v\cdot z^{\nu_1+\nu_2}=(e^v\cdot z^{\nu_1})(e^v\cdot z^{\nu_2})$.
- Open toric orbits <-> cones in fan $\mathcal{O}_\tau\subset U_\tau$. (take $\{z^\nu=0\ \forall\nu\notin\sigma^\perp\}$)
- The exact sequence $0\to K\to\mathbb{Z}^m\to N\to 0$.
- Ex: what is the fan for blow-up of C^3?





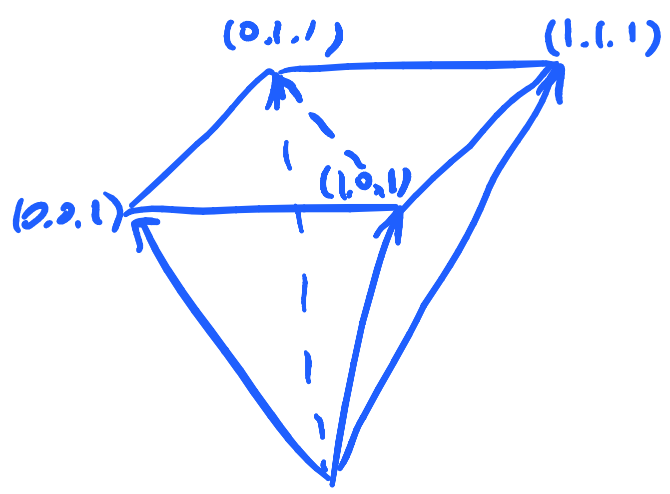


$\mathcal{O}(-1) \oplus \mathcal{O}(-1)$

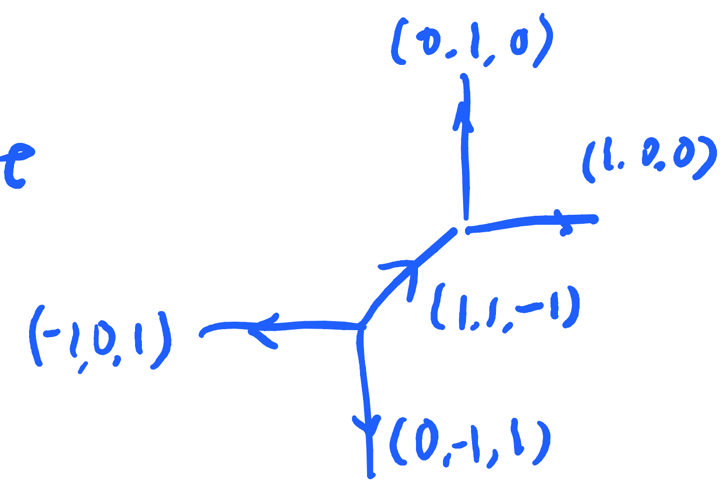
$\downarrow$ blow-up

conifold

fan



polytope



conifold:

$$\mathrm{Spec}\left(\mathbb{C}[X,Y,Z,W] / \langle XZ - YW \rangle\right).$$