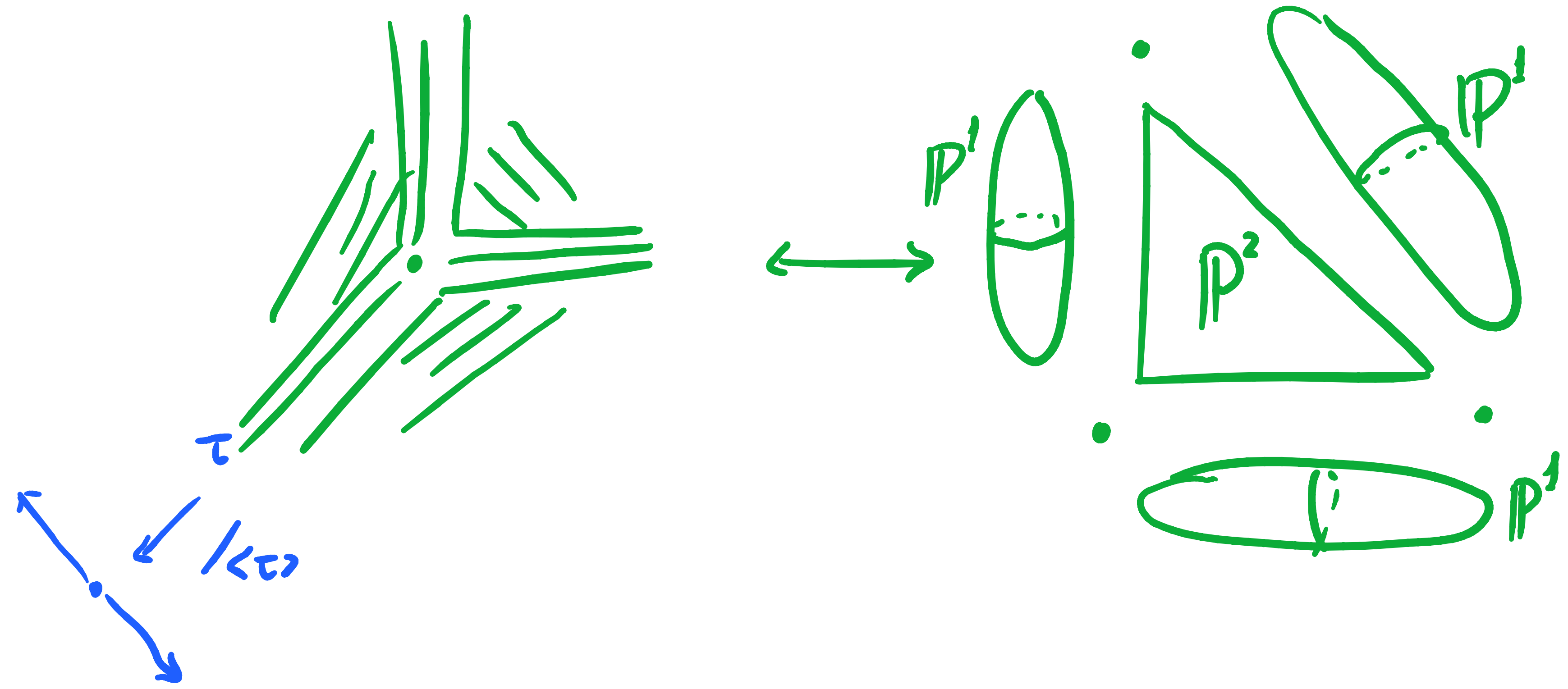
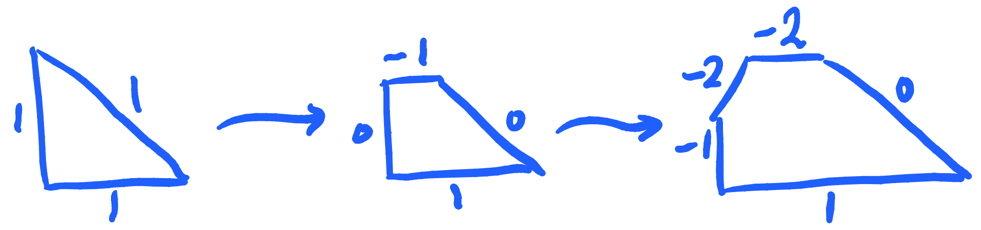
Sunday, January 17, 2016 9:13 PM

- Simply-connected if fan has top dimensional cone
- Euler characteristic = # top-dim cones
- Toric subvarieties; generate cohomology $\tau \longleftrightarrow \overline{O}_\tau = \{z^\nu = 0 \ \forall \nu \notin \sigma^\perp\}$
- Weil divisor: codimension-one subvarieties. Divisor line bundles
- Toric Weil divisors <-> rays of fan
- Toric meromorphic functions <-> M $\exp 2\pi i(\nu,\cdot)$ on $N_{\mathbb{C}}/N$.
- Principle divisors; linear equivalence $\sum (\nu, v_i) D_i$.
- Computing intersection numbers, eg F_n
- Cech cohomology by toric charts Fact: $H^{>0}(F) = 0 \ \forall O_x$-sheaf F/aff. var.
- Ex. compute the self-intersection numbers of toric curves in blow-ups.

Can always use C^* action to push any cycle to be contained in a toric divisor

toric: $(N, \Sigma_\tau)/\langle\tau\rangle$.





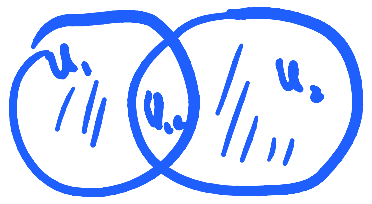
Cech: F : sheaf of gp.

$\mathcal{U} = \{U_i\}$ open cover.

$C^k(F) \ni \{s_I \in F(U_I) : |I| = k+1\} \triangleq s.$ w/ $s_I = \operatorname{sgn}(\pi) s_{\pi(I)} \ \forall$ permut. π

$\downarrow \delta$

$C^{k+1}(F) \ni \delta \cdot s$ on $U_J = \sum_{i=0}^{k+1} (-1)^{i+1} s_{(j_0 \cdots \hat{j}_i \cdots j_{k+1})}\Big|_{U_J}$. $|J| = k+2$



$\xrightarrow[\text{coho.}]{} \check{H}(\mathcal{U}, F)$.

$\check{H}(F) \triangleq \varinjlim_{\mathcal{U}} \check{H}(\mathcal{U}, F)$.

(need refinement of $\mathcal{U}$.)

sheaf: open set $\longrightarrow$ group (of loc. sect.)

have restriction $F_U \to F_V$ for $U \supset V$.

(= id when $U = V$; respect composition)

loc. determine global:

$$\begin{cases} s|_{U_i} = t|_{U_i} \ \forall i, \ \bigcup_i U_i = U \Rightarrow s = t \text{ on } U. \\ s_{U_i} \text{ w/ } s_{U_i}|_{U_i \cap U_j} = s_{U_j}|_{U_i \cap U_j} \Rightarrow s \text{ on } \bigcup U_i. \end{cases}$$

e.g. $\Omega^\cdot, \mathcal{O}, \mathcal{O}(E), \mathbb{Z}, \mathbb{R}$.

e.g. $\mathcal{O}(k)$ $\quad (e_1 = z_2^k e_2.)$

$\downarrow$

$\mathbb{P}^1$. $\quad U_1 \ \mathbb{C}\ 0 \quad U_2 \ \mathbb{C}$

$H^0 = \{\text{global sections}\} = \mathbb{C}^{\max\{k,0\}}$.

$H^1 = \{\text{sections on } U_{12}\} / \underbrace{\langle \sigma_1 - \sigma_2 \rangle}$

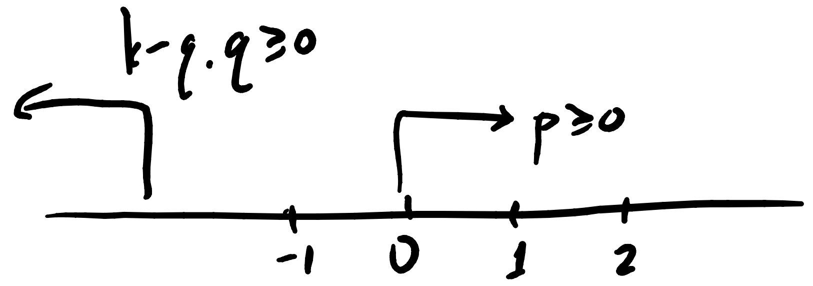
$z_1^p e_1 \ , \ z_2^q e_2 \quad p, q \geq 0$

$\qquad\qquad \| $

$\qquad z_2^{q-k} e_1$

$\qquad\qquad \| $

$\qquad z_1^{k-q} e_1$



if $k \geq 0$. $z^\ell \sim 0 \ \forall \ell \Rightarrow H^1 = 0$.

if $k < 0$, $z_1^{-1}, \dots, z_1^{k+1}$ remained.

$\therefore H^1(\mathcal{O}(-k)) \simeq H^0(\mathcal{O}(k-2))$

(Serre duality: $H^p(\mathcal{F}) = (H^{n-p}(K \otimes \mathcal{F}^*))^*$.)

(Note: $H^0(\mathcal{O}(-1)) = H^1(\mathcal{O}(1)) = 0$!)