
We have seen $T^*B/\Lambda^* \cong TB/\Lambda$.

The closest geometry to this is $X_\Delta$: toric fiber.

Ignoring sing. fibers $(T^*X_\Delta, \omega) \cong (C^*)^n$.

Motivation.

\[ T^*X \xrightarrow{\mu} P \subset \mathbb{C} \]

\(\text{e.g.} \)

\[ \text{area of line } l = a. \]

\[ \text{area of } \beta(u,v) = a-r_i-r_j. \]

\[ \text{linear wind. } \chi \in T^*P \text{ is action wind. on } P^0: \]

\[ \int_{T^*X} \nu \omega = \int_0^1 \tilde{\omega}_x(v_x(u), X \cdot v_x(u)) \, dt \]

\[ (v \in T_xP) \]

\[ = -\int_0^1 v_x(u) \cdot \mu^X \, dt = -v \cdot X. \]

This is Lag. Floer theory applied on fibers.

(Pho–Oh–Pao, Food general.)
Discs and Log. fiber.

\((X_\Delta, \omega) \Rightarrow F_r\)

\(\Delta \ni \tau\)

Consider \(H_2(X, F_r)\).

Two important classical symplectics inv. ass. to \(\beta \in H_2(X, F_r)\):

- sympl. area: \(\int_{\beta} \omega\)
- Masur index: \(\Theta \mapsto \left[ Gr \to u(\beta) \right] \in \Omega_1^{\infty}(H_2, \mu)\)

\(\frac{\partial u(s)}{\partial s}\)

If \(\rho \in X\), then \(\mu(s) \in 2F_r\).

\(\mu\) is important for computing disc. of moduli.
It is an analog of \(\alpha(u)\) for \(u \in H_2(X)\).

\(\text{e.g. } \rho'\)

\(\text{area of } \rho' = a\).

\(\text{area of } \beta = a-r\).

\(\text{ex. } \int_{F_r}\)

\((a, b) \quad (a, b)\)

- What is the area of \(f\) and \(s \in H_2(X)\)?
- Compute the disc potential.
- Find a non-displaceble Log. fiber.
- What are the areas of the basic discs bad by the non-displaceble Log. fiber?
Minor space \( TP^o / \Lambda \subset (\mathbb{C}^*)^n \)  
\{ flat unit connections \( \nabla \) on fiber at \( r \in P^o \) \}

Complex coord. \( Z_k = \exp (-r_k + i \theta_k) \)  
on minor  
true action coord. 

\[
= \exp (-\text{area}(\bigcirc_{k \neq 0} \cdots) \cdot \text{Hd}_{\Omega} (Y_k) .
\]

For \( \beta \in H_2(X, \mathcal{L}_r) \),  
\[
\exp (-\text{area}(\beta(r)) \cdot \text{Hd}_{\Omega} (\partial \beta) \text{ monomial in } Z_i .
\]
Hol. disc in tone case

In this case, \( \mu(x) = \beta_x \cdot (D_{m+n} + D_m) \).

Hol. map \((D, 0) \rightarrow (X, L)\) is the unit circle

Smooth hol. discs in a tone field is classified by \([C^0_{[0, \infty]} \cup [0, \infty])\) (unit. tone Kord str.).

E.g. \(C^0\). Consider \( f : (\Delta, \partial \Delta) \rightarrow (C^n, F_n) \).

\( \Delta = f^2(1) \) must be interior points

by identity. \( f \) is cont. by max. principle (if \( f \) is cont)

Otherwise, ind. \( f \) is cont. and is so cont. \( C_n \).

\( f_\infty = \int \frac{e^{-x^2}}{1-x^2} \).

\( f = e^{-x^2} \).

Example, unit circle \((\Delta, \partial \Delta)\).

\[ \exp(-a \cdot \text{diag}(\beta_i \cdot r_i)) \cdot H_{a/2}(d \beta_i) = e^{-i \theta} \cdot z \cdot z^{-1} = z \cdot z^{-1}. \]

\[ \exp(-a \cdot \text{diag}(\beta_i \cdot r_i))^{-1} = \frac{z \cdot z^{-1}}{2}. \]
Basic disc class in $H^2(X, L_r)$:

- Take $\nu$: primitive generator of a ray of $\Sigma$.
- \( N = \pi_1(L_r) \).
- Take a path connecting $r$ to a point in $D_0$.
- Move $Y_r$ along the path
- and $Y_r$ vanishes at $D_0 \rightarrow$ disc $f_r$.

\[ W = \sum_r \mathbb{R} \mathbb{P}^r \text{ for toric Fano manifold}. \]
Fukaya-Oh-Ohta-Ono:

Critical points of $W$ corr. to non-displaceable fibers!

Eg.

\[ W = z + \frac{q}{z} \]

\[ \frac{\partial W}{\partial z} = 0 \iff z = \pm \sqrt{q} \]

\[ = \exp\left(-\frac{q}{2}\right) e^{i\theta}, \theta = 0, \pi. \]

Eg.

\[ W = z + w + \frac{q}{2z} \]

Crit. pt. \( p_3 \): \( z = w = q^{\frac{1}{3}} \)

\[ = \exp\left(-\frac{q}{3}\right) e^{i\theta}, \theta = 0, \pm \frac{2\pi}{3}. \]
Measures of homomorphic discs (to count discs)

- We always use the toric eqn. tr. $J$.

The $\Sigma$-$\Phi$. theory is indep. of the choice of
almost eqn. trs. (and partition in virtual theory).

\[ \text{(up to An. sim.)} \]

For $\beta \in \gamma_1(\Sigma, \Phi)$

\[ M_{\text{red}}(\beta) = \left\{ (\Sigma, \Phi \to (X, L), \beta, \tau_1, \tau_2, \ldots, \tau_k) \right\}_{\text{stable}} / \text{Aut}(\Sigma) \]

- intersection eqn. model.
- $J$-ful. on each cut.
- $\beta \vdash \Sigma_0$, $\ldots$, $\Sigma_k$.
- $\beta$ are uniquely oriented.
- stable (not finite): If $u$ is non-vert. on a cut,
  then $2n_u + n_\beta \geq 3$ for this cut.
  $N_{\text{cut}} = 3$ for sphere cut.
  $L = (\text{int. mod.} = \text{red tr.})$.

Have eqn.:

\[ M_{\text{red}}(\beta) \to X^L \cdot L^L \]
Spontaneous \( \langle \theta \rangle \).

Recall \( \frac{T \Delta \theta}{N} \times \frac{T \Delta \theta}{M} = \chi \).

\( X_0 \) \( \{ (F_0, \varphi) : \Re, \Delta \varphi; \ \varphi \text{ flat only} \} \).

Opt tene.

\( \varphi \) \( \text{side for } \varphi \neq \infty \).

Gx cond on \( X \) is \( \text{adjacent fields are connected by} \).

\[ z_i = \frac{e^{i k \omega} H \phi_0(x_0)}{e^{-k \omega} e^{i \Phi}} \]

SYG mirror obtained by pushing \( \Phi, W_1^I : X \rightarrow C \),

\[ W_1^I = \sum_{p \text{ paths}, \varphi} \sum_{x_0} \Phi \frac{e^{i k \omega} H \phi_0(x_0)}{e^{-k \omega} e^{i \Phi}} \]

\[ \frac{\text{octoh.}}{\text{octoh.}} \sum_{x_0} e^{i k \omega} H \phi_0(x_0). \]

Using \( f_i = -v_i \) as basis,

\[ \Phi = \frac{\delta}{v_0} \sum_i v_i \phi_i \]

(for \( i = 1 \ldots \), \( v_{\infty} = \delta_{\infty} \).

Thus \( \Phi = \sum_i \delta_i v_i \phi_i \in \mathcal{H}_i(x) \), \( (\delta_i = 0 \text{ for } i = 1 \ldots \).

Metal \( \psi \rightarrow W_1^{\psi} = \sum_{i=1}^n e^{i k \omega} v_i \phi_i \in \mathcal{H}_i \psi \).

Hermitian potential \( \rightarrow W_1^{\psi} = \sum_{i=1}^n e^{i k \omega} v_i \phi_i \in \mathcal{H}_i \psi \).
\[ W = z + z + \frac{1}{z} \]
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For any toric variety,
\[ W^{HV} = \sum_{i=1}^{N} C_{-k\omega} \omega \frac{1}{2} \]

\[ ] \text{depends on rays of the fan.} \]
\[ ] \text{(eg. does not distinguish flop.)} \]
\[ \text{loc.} \quad \rightarrow \quad \]
\[ \text{fan:} \quad \]
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