MA122 In-class Practice Problem Set 2

(1) Find the areas bounded by the indicated equations:

  (a) $y = -x(3 - x); y = 0; 1 \leq x \leq 2$
  Answer: 2.167

  (b) $y = x^2 + 1; y = 2x - 2; -1 \leq x \leq 2$
  Answer: 9

  (c) $y = -\sqrt{100 - x^2}; y = \sqrt{100 - x^2}; -10 \leq x \leq 10$
  Answer: 314.159

  (d) $y = x^3 + 1; y = x + 1$
  Answer: 0.5

(2) Gini Index:
A country is planning changes in tax structure in order to provide a more equitable distribution of income. The two Lorenz curves are: $f(x) = x^2.3$ currently, and $g(x) = 0.4x + 0.6x^2$ proposed. Will the proposed changes work?
Answer: 0.3939, 0.20, The Gini index is decreasing, so the future distribution will be more equitable.

(3) Probability:
In a certain city, the daily use of water in hundreds of gallons per household is a continuous random variable with probability density function

$$f(x) = \begin{cases} 0.15e^{-0.15x}, & x \geq 0; \\ 0, & \text{otherwise} \end{cases}$$

Find the probability that a household chosen at random will use between 300 and 600 gallons.
Answer: $-e^{-0.9} + e^{-0.45} \approx 0.23$

(4) Drug assimilation:
The rate at which the body eliminates a certain drug (in milliliters per hour) is given by

$$R(t) = \frac{60t}{(t+1)^2(t+2)}$$

where t is the number of hours since the drug was administered. How much of the drug is eliminated during the first hour after it was administered? During the fourth hour?
Answer: 4.522mL; 1.899mL

(5) Continuous Income Stream:
Find the total income produced by a continuous income stream in the first 2 years if the rate of flow is

$$f(t) = 600e^{0.06t}$$

Answer: 1275 dollars

(6) Future Value of a Continuous Income Stream:
Lets continue the previous example where

$$f(t) = 600e^{0.06t}$$

Find the future value in 2 years at a rate of 10%.
Answer: 1408.59 dollars

(7) Consumers' Surplus:
Find the consumers' surplus at a price level of $\bar{p} = 120$ for the price-demand equation

$$p = D(x) = 200 - 0.02x$$

Answer: $\bar{x} = 4,000, CS = 160,000$ dollars

(8) Producers' Surplus:
Find the producers' surplus at a price level of $\bar{p} = 55$ for the price-supply equation

$$p = S(x) = 15 + 0.1x + 0.003x^2$$

Answer: $\bar{x} = 100, PS = 2,500$ dollars

(9) Find:

(a) $\int x^3 \ln x dx$
Answer: $\frac{x^4}{4} \ln x - \frac{x^4}{16} + C$

(b) $\int x^3 e^x dx$
Answer: $(x^3 - 3x^2 + 6x - 6)e^x + C$

(c) $\int_0^1 \ln e^{x^2} dx$
Answer: $\frac{1}{3}$

(10) Use table of integrals to find:

(a) $\int \frac{1}{\sqrt{x+16}} dx$
Answer: $\frac{1}{4} \ln \left| \frac{\sqrt{x+16}-4}{\sqrt{x+16}+4} \right| + C$

(b) $\int x^2 \sqrt{9x^2 - 1} dx$
Answer: $\frac{1}{72} 3x\sqrt{18x^2 - 1}\sqrt{9x^2 - 1} - \frac{1}{216} \ln |3x + \sqrt{9x^2 - 1}| + C$

(c) $\int x^3 e^{-x} dx$
Answer: $-x^3e^{-x} - 3x^2e^{-x} - 6xe^{-x} - 6e^{-x}C$