MA122 In-class Practice Problem Set 4

(1) Find the derivatives of $f(x)$:

(a) $f(x) = \cos(x^3 + 9x)$
Answer: $-(3x^2 + 9)\sin(x^3 + 9x)$

(b) $f(x) = \sin^8(x^3)$
Answer: $24x^2 \sin^7(x^3)\cos(x^3)$

(2) Find the indefinite integrals:

(a) $\int \sin(-25x)dx$
Answer: $\frac{1}{25}\cos(-25x) + C$ or $\frac{1}{25}\cos(25x) + C$

(b) $\int \frac{\cos x}{\sqrt{\sin x}}dx$
Answer: $2\sqrt{\sin x} + C$

(c) $\int x^2 \cos(x^3)dx$
Answer: $\frac{\sin(x^3)}{3} + C$

(d) $\int (x + 1)\cos(x^2 + 2x)dx$
Answer: $\frac{1}{2}\sin(x^2 + 2x) + C$

(3) Find the definite integrals:

(a) $\int_0^{\pi/4} \cos x dx$
Answer: $\frac{\sqrt{2}}{2}$

(b) $\int_0^{\pi/2} \cos x dx$
Answer: 1

(c) $\int_{\pi/2}^{\pi} \cos x dx$
Answer: -1

(d) $\int_0^{\pi} \cos x dx$
Answer: 0

(e) $\int_{\pi/6}^{\pi/3} \cos x dx$
Answer: $\frac{-1+\sqrt{3}}{2}$

(f) $\int_{\pi/2}^{2\pi/3} \cos x dx$
Answer: $\frac{\sqrt{3}-2}{2}$

(4) Find the trigonometric integrals:

(a) $\int \sin^8 x \cos^3 x dx$
Answer: $\sin^9 x/9 - \sin^{11} x/11 + C$

(b) $\int \sin^4 x dx$
Answer: $\frac{3}{8}x - \frac{\sin(2x)}{4} + \frac{\sin(4x)}{32} + C$

(c) $\int \tan^6 x \sec^4 x dx$
Answer: $\tan^9 x/9 + \tan^7 x/7 + C$

(d) $\int \cot^6 x \csc^4 x dx$
Answer: $-\cot^9 x/9 - \cot^7 x/7 + C$

(e) $\int \tan^3 x \sec^5 x dx$
Answer: $\sec^7 x/7 - \sec^5 x/5 + C$

(f) $\int \cot^3 x \csc^5 x dx$
Answer: $-\csc^7 x/7 + \csc^5 x/5 + C$

(g) $\int \frac{1}{\sin^4 x \cos^2 x} dx$
Answer: $\tan x - 2\cot x - \frac{\cot^3 x}{3} + C$

(h) $\int \frac{3\sin x + 4\cos x}{\sin x + 2\cos x} dx$
Answer: $\frac{5}{11}x - \frac{2}{5}\ln|\sin x + 2\cos x| + C$

(5) A soft-drink company has revenues from sales over a 2-year period as given approximately by

$$R(t) = 4 - 3\cos\left(\frac{\pi t}{6}\right) \quad 0 \le t \le 24$$

where $R(t)$ is revenue (in millions of dollars) for a month of sales t months after February 1.

(a) What is the total revenue taken in over the 2-year period?
Answer: 96 million dollars.

(b) What is the total revenue taken in from $t = 6$ to $t = 9$?
Answer: 17.73 million dollars.

(6) Find

$$I = \int e^{2x} \sin(3x) dx$$

Answer: $\frac{1}{13}e^{2x}[2\sin(3x) - 3\cos(3x)]$