MA122 Practice Problem Set 6

- **Taylor polynomials**

1. (a) Find the 7-th degree Taylor polynomial at $x = 0$ for $f(x) = \sin x$
   Answer: $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}$

   (b) Find the $(2n+1)$-th degree Taylor polynomial at $x = 0$ for $f(x) = \sin x$
   Answer: $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots + \frac{(-1)^n x^{2n+1}}{(2n+1)!}$

   (c) Find the 6-th degree Taylor polynomial at $x = 0$ for $g(x) = \cos x$
   Answer: $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}$

   (d) Find the $2n$-th degree Taylor polynomial at $x = 0$ for $g(x) = \cos x$
   Answer: $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots + \frac{(-1)^n x^{2n}}{(2n)!}$

2. – Use the second order Taylor polynomial $p_2(x)$ of $f(x)$ to approximate the given quantities.
   – Compute the absolute error in the approximation assuming the exact value is given by a calculator.

   (a) $f(x) = \sqrt[3]{1+x}$. Approximate $\sqrt[3]{1.1}$.
   Answer: $p_2(0.1) \approx 1.032$, the absolute error $\approx 5.79 \times 10^{-5}$

   (b) $f(x) = \ln(1+x)$. Approximate $\ln(1.06)$.
   Answer: $p_2(0.1) \approx 0.0582$, the absolute error $\approx 6.89 \times 10^{-5}$

   (c) $f(x) = e^{-x}$. Approximate $e^{-0.15}$.
   Answer: $p_2(0.1) \approx 0.86125$, the absolute error $\approx 5.42 \times 10^{-4}$

3. Match the following six functions with the given six Taylor polynomials of order 2.

a. $\sqrt{1+2x}$

b. $\dfrac{1}{\sqrt{1+2x}}$

c. e^{2x}

d. $\dfrac{1}{1+2x}$

e. $\dfrac{1}{(1+2x)^3}$

f. e^{-2x}

A. $p_2(x) = 1 + 2x + 2x^2$

B. $p_2(x) = 1 - 6x + 24x^2$

C. $p_2(x) = 1 + x - \dfrac{x^2}{2}$

D. $p_2(x) = 1 - 2x + 4x^2$

E. $p_2(x) = 1 - x + \dfrac{3}{2}x^2$

F. $p_2(x) = 1 - 2x + 2x^2$

Answer: a-C, b-E, c-A, d-D, e-B, f-F