Formula Sheet

- Integrals:

$$\int x^n \, \mathrm{d}x = \frac{1}{n+1} x^{n+1} + C, \quad n \neq -1 \qquad \int e^x \, \mathrm{d}x = e^x + C$$

$$\int \frac{1}{x} \, \mathrm{d}x = \ln|x| + C \qquad \int \cos x \, \mathrm{d}x = \sin x + C$$

$$\int \sqrt{a^2 + x^2} \, \mathrm{d}x = \frac{1}{2} x \sqrt{a^2 + x^2} + \frac{1}{2} a^2 \ln\left(x + \sqrt{a^2 + x^2}\right) + C$$

- Arc Length & Area & Volume:

  (a) Arc Length:

  $$L = \int_a^b \sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2} \, \mathrm{d}t$$

  where $x = f(t)$ and $y = g(t)$, $a \leq t \leq b$.

  (b) Area:

  $$A = \int_a^b \left[y_T - y_B\right] \mathrm{d}x \quad \text{or} \quad \int_c^d \left[x_R - x_L\right] \mathrm{d}y$$

  (c) Volume:

  (i) Disk and washer method:

  $$V = \int_a^b A(x) \, \mathrm{d}x \quad \text{or} \quad \int_c^d A(y) \, \mathrm{d}y$$

  where $A(x)$ or $A(y)$ is the cross-sectional area.

  (ii) Cylindrical shell method:

  $$V = \int [\text{circumference}][\text{height}][\text{thickness}]$$

- Maclaurin series:

$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \ldots$	$R = 1$
$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$	$R = \infty$
$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots$	$R = \infty$
$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots$	$R = \infty$
$\arctan x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots$	$R = 1$
$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots$	$R = 1$