Boston University
MA 581
Probability, Discussion 2
Summer 2, 2013

1 Conditional Probability

1. Out of n lottery tickets, there is only one winning.

   (a) Suppose that the first $k-1$ people fail to win, what is the probability that the k-th person wins the lottery.

   (b) Find the probability that the k-th person wins the lottery.

2. Suppose that the proportion of rainy days out of a year in Boston and South Hadley are 20% and 18%, respectively. While 12% of the times, it rains in both Boston and South Hadley.

   (a) Ian lives in Boston and Wes lives in South Hadley. One day, Ian saw Wes's posts of the Mount Holyoke in the rain on facebook (so Ian knew that it rained in South Hadley). What is the probability that it also rained in Boston on that day.

   (b) Find the probability that it rains in Boston or South Hadley.

2 Law of Total Probability

1. Consider a red balls and b blue balls. We draw balls one by one randomly without replacement. Find the probability that the second ball is a red one.

2. Hunter buys watermelons from the Star Market everyday in the summer and he always uses the app "iWatermelon" which provides an estimate of the watermelons when you hit them on the surface. Hunter tries the app on all the watermelons in stock. "iWatermelon" tells Hunter that 90% are excellent, 5% are good, 3% are fair and 2% are bad. According to the ratings of the app, it is believed that the probability of the watermelons being truly super sweet is $50\%, 20\%, 5\%$ and 1% when "iWatermelon" shows "amazing", "good", "fair" and "bad".

   (a) What is the probability of the watermelon that Hunter buys is a sweet one.

   (b) Hunter ate a watermelon the night he bought it and found that it's a super sweet one. But he forgot what "iWatermelon" said on that watermelon. What is the probability that "iWatermelon" told Hunter"amazing" when he tried it on that watermelon.

3. (*being optimistic when diagnosed with cancer*) Suppose that the proportion of liver cancer patients is 0.04%. A medical center adopts a technique to detect cancer. The medical

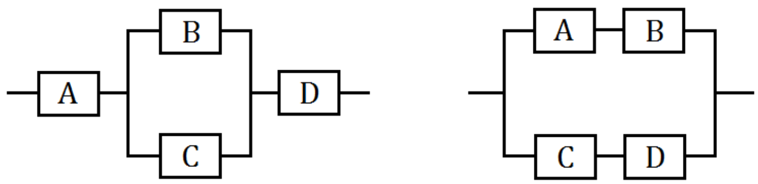

Figure 1: Two Systems

center tests the precision of the technique on known liver cancer patients and normal patients. The proportion of positive reports on the patient truly has live cancer is 95%, and that of negative reports on patients do not have liver cancer is 90%. If a patient is diagnosed with liver cancer using that technique, what is the probability that the patient truly has liver cancer.

3 Independence

1. We draw two balls one by one randomly from a red balls and b blue balls. Let event $A = \{\text{first one is red}\}$ and event $B = \{\text{second one is red}\}$. Are A and B independent? Consider the case:

   (a) with replacement

   (b) without replacement

2. Consider a tetrahedron with three faces colored red, green and blue respectively and the other face has all three colors. Roll the tetrahedron randomly. Let events $A = \{\text{subface has red}\}$, $B = \{\text{subface has green}\}$ and $C = \{\text{subface has blue}\}$. Are A, B and C mutually independent?

3. Show that if events A and B are independent, so are A^c and B (A and B^c, A^c and B^c).

4. Let events $A_1, \ldots, A_n$ be mutually independent. $P(A_i) = p_i$, $i = 1, \ldots, n$. Find the probability that:

   (a) none of the events occurs.

   (b) at least one event occurs.

   (c) exactly one event occurs.

5. Components A, B, C and D are mutually independent and each works normally with probability p ($p > 0$). Which of the two systems in Figure 1 is more reliable?