Boston University
MA 581
Probability, Discussion 3
Summer 2, 2013

PMF of a Discrete Random Variable

1. Find c such that $P(X = k) = c\lambda^k/k!$, $k = 1, 2, \ldots,$, $\lambda > 0$ is a PMF.

2. A points moves along a line to the right and the left with probability p and $1 - p$, respectively, at each unit of time. It starts from the origin at time 0. Let X_n be the number of right moves until time n and Y_n be the position of the point at time n. Find the PMF of X_n and Y_n.

3. (a) Consider a bag with 5 balls numbered by 1, 2, 3, 4, 5. Draw 3 balls randomly from the bag at one time. Let X be the maximum number on the balls and Y be the minimum number on the balls. Find the PMF of X and Y, respectively.

   (b) Members of a population are numbered $1, 2, \ldots, N$. A random sample of size n is taken without replacement. Determine the PMF of (1) the largest numbered member obtained; (2) the smallest numbered member obtained

   (c) Repeat (b) if the sample is taken with replacement.

4. X is a Poisson Random Variable. $p_X(1) = p_X(2)$. Find $p_X(4)$.

Joint and Conditional PMF

1. Refer to the example about two electric components in lecture, where the joint PMF of X and Y is

$$p_{X,Y}(x, y) = p^2(1 - p)^{x+y-2}, x, y \in \mathcal{N}$$

   and $p_{X,Y}(x, y) = 0$ otherwise, where $0 < p < 1$.

   (a) Determine $P(X = Y)$

   (b) Use FPF to determine $P(X > Y)$. Interpret your answer.

   (c) Without doing any computations, explain why $P(X > Y) = P(X < Y)$

   (d) Use the results of (a) and (c) to obtain $P(X > Y)$.

2. (a) Let X and Y be independent Poisson random variables, $X \sim \text{Pois}(\lambda_1)$ and $Y \sim \text{Pois}(\lambda_2)$. Determine and identify the PMF of $X + Y$.

   (b) Determine and identify the conditional PMF of $X|(X + Y)$.

   (c) Can you generalize (a) and (b) to the case where there are n independent Poisson random variables? Why or why not.

3. (a) (*Combination*) Explain that $\binom{m+n}{z} = \sum_{x=0}^{m} \binom{m}{x}\binom{n}{z-x}$.

   (b) Let X and Y be independent random variables, $X \sim \text{Bin}(m, p)$ and $Y \sim \text{Bin}(n, p)$. Determine and identify the PMF of the random variable $X + Y$.

   (c) Determine and identify the conditional PMF of $X|(X + Y)$.

   (d) Can you generalize (b) and (c) to the case where there are n independent Binomial random variables with the same success probability p? Why or why not.