MA 581 Final, Summer 2013
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You may use two 2-sided 3×5 note cards to assist you. Make sure to write all of your work and solutions in your blue book. You will not get credit for anything other than what is in your blue book. GOOD LUCK!

Problem 1 *(10 points) The lifetime of a component in an electronic device has an exponential distribution with* ***median*** *of 4 hours.*

Calculate the probability that the component will work without failing for at least five hours.

Problem 2 *(15 points) Let* $X \sim \Gamma(\alpha, \lambda)$.

1. *Find the MGF of X.*
2. *Use the MGF to find Var(X).*

Problem 3 *(15 points) Suppose that* $X \sim Pois(\lambda)$.

1. *For* $n \in \mathcal{N}$, *determine* $E\big(X(X-1)\cdots(X-n+1)\big)$, *called the* n*th factorial moment.*
2. *Use part 1 to find* $E(X)$ *and* $Var(X)$.

Problem 4 *(10 points) Suppose that the joint pdf of* X *and* Y *is given by*

$$f_{X,Y}(x,y) = \frac{e^{-x/y}e^{-y}}{y} \quad x > 0, y > 0$$

and 0 otherwise. Find $P(X > 2|Y = 3)$.

Problem 5 *(10 points) Suppose that by any time* t, *the number of people that have arrived at the Marsh Plaza BU shuttle stop is a Poisson random variable with mean* λt. *It the initial shuttle arrives at Marsh Plaza stop at a time that is uniformly distributed over* $(0, T)$. *What is the mean and variance of the number of passengers that enter the shuttle?*

Problem 6 *(15 points) Let* $X \sim Exp(\alpha)$ *and* $Y = \beta e^X$, *where* β *is a positive real number and* $\alpha > 1$.

1. *Obtain the pdf of* Y *and use* $f_Y(y)$ *to determine* $E(Y)$.
2. *Determine* $E(Y)$ *by using the FEF (by using the pdf of* X*).*
3. *Determine* $E(Y)$ *by using the tail probabilities of* Y. *(*$E(Y) = \int_0^\infty P(Y > y)dy$ *if* Y *is a nonnegative continuous random variable.)*

Problem 7 *(15 points) If X and Y are two independent gamma random variables with parameters (α, λ) and (β, λ), respectively.*

1. *Compute the joint density of $U = X + Y$ and $V = \frac{X}{X+Y}$.*
2. *Identify the distribution of U and V, respectively.*
3. *Show that U and V are independent.*

***(Extra Credit** 10 points) Let X and Y be independent and identically distributed uniform random variables on (a, b), $0 < a < b$. Find the pdf of $X + Y$ and $\frac{Y}{X}$, respectively.*