MA 581 Final Formula Sheet

Bernoulli Distribution: $X \sim Bern(p)$

$$p_X(x) = \begin{cases} p^x(1-p)^{1-x} & \text{if } x = 0, 1 \\ 0 & \text{otherwise} \end{cases}, \quad E(X) = p, \quad Var(X) = p(1-p)$$

Binomial Distribution: $X \sim Bin(n,p)$

$$p_X(x) = \begin{cases} \binom{n}{x} p^x(1-p)^{n-x} & \text{if } x = 0, 1, ..., n \\ 0 & \text{otherwise} \end{cases}, \quad E(X) = np, \quad Var(X) = np(1-p)$$

Hypergeometric Distribution: $X \sim H(N,n,p)$

$$p_X(x) = \begin{cases} \frac{\binom{Np}{x}\binom{N(1-p)}{n-x}}{\binom{N}{n}} & \text{if } x = 0, 1, ..., n \\ 0 & \text{otherwise} \end{cases}, \quad E(X) = np, \quad Var(X) = \left(\frac{N-n}{N-1}\right) np(1-p)$$

Poisson Distribution: $X \sim Pois(\lambda)$

$$p_X(x) = \begin{cases} e^{-\lambda}\frac{\lambda^x}{x!} & \text{if } x = 0, 1, ... \\ 0 & \text{otherwise} \end{cases}, \quad E(X) = \lambda, \quad Var(X) = \lambda$$

Geometric Distribution: $X \sim Geo(p)$

$$p_X(x) = \begin{cases} p(1-p)^{x-1} & \text{if } x = 1, 2, ... \\ 0 & \text{otherwise} \end{cases}, \quad E(X) = 1/p, \quad Var(X) = (1-p)/p^2$$

Negative Binomial Distribution: $X \sim NB(r,p)$

$$p_X(x) = \begin{cases} \binom{x-1}{r-1} p^r(1-p)^{x-r} & \text{if x = r, r+1, ...} \\ 0 & \text{otherwise} \end{cases}, \quad E(X) = r/p, \quad Var(X) = r(1-p)/p^2$$

Uniform: $X \sim U(a,b)$

$$f_X(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \text{otherwise} \end{cases}, \quad E(X) = (a+b)/2, \quad Var(X) = (b-a)^2/12$$

Exponential: $X \sim Exp(\lambda)$, $\lambda > 0$

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & x > 0 \\ 0 & \text{otherwise} \end{cases}, \quad E(X) = 1/\lambda, \quad Var(X) = 1/\lambda^2$$

Normal: $X \sim N(\mu, \sigma^2)$, $\sigma^2 > 0$

$$f_X(x) = \begin{cases} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} & -\infty < x < \infty \\ 0 & \text{otherwise} \end{cases}, \quad E(X) = \mu, \quad Var(X) = \sigma^2$$

Gamma: $X \sim \Gamma(\alpha, \lambda)$, $\alpha, \lambda > 0$

$$f_X(x) = \begin{cases} \frac{\lambda^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\lambda x} & x > 0 \\ 0 & \text{otherwise} \end{cases}, \quad E(X) = \alpha/\lambda, \quad Var(X) = \alpha/\lambda^2$$

Recall that $\Gamma(\alpha) = (\alpha - 1)!$ if α is a positive integer.

Beta: $X \sim Beta(\alpha, \beta)$, $\alpha, \beta > 0$

$$f_X(x) = \begin{cases} \frac{1}{B(\alpha,\beta)} x^{\alpha-1}(1-x)^{\beta-1} & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}, \quad E(X) = \alpha/(\alpha+\beta), \quad Var(X) = \alpha\beta/(\alpha+\beta)^2(\alpha+\beta+1)$$

Recall that $B(\alpha, \beta) = \int_0^1 x^{\alpha-1}(1-x)^{\beta-1}dx$ and $B(\alpha, \beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$ for $\alpha > 0, \beta > 0$.