

MA 581 Exam 1, Summer 2013
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The following exam is 7 questions for a total of 90 points. You may use a 2-sided 3×5 notecard to assist you. Make sure to write all of your work and solutions in your blue book. You will not get credit for anything other than what is in your blue book. You are not allowed to use calculators. You may leave your answers with the binomial coefficients. GOOD LUCK!

Problem 1 (10 points) Find c such that $P(X = k) = c\lambda^k/k!$, $k = 2, 3, \dots$, $\lambda > 0$ is a PMF.

Problem 2 (10 points) A point moves along a line to the left and the right with probability p and $1 - p$, respectively, at each unit of time. It starts from the origin at time 0. Let X_n be the number of left moves until time n and Y_n be the position of the point at time n . Find the PMF of X_n and Y_n .

Problem 3 (15 points) In your local state lottery, a drawing is held each week where 5 numbers between 1 and 37 are chosen randomly, without replacement and their order does not matter. Before the drawing, you can purchase a ticket on which you guess 5 numbers (also between 1 and 37, without replacement, order does not matter).

1. What is the probability of guessing exactly 3 numbers from the drawing correctly?
2. Let X be the number of people among 10^6 people guessing all 5 numbers correctly. Identify the distribution of X .
3. What is the probability of 2 or more people among 10^6 people guessing all 5 numbers correctly?

Problem 4 (15 points) Draw two numbers at random from $\{1, 2, 3, 4\}$ without replacement. Let X denote the smaller of the two numbers and Y denote the larger of the two numbers.

1. Make a table which contains the joint probability mass function $p_{X,Y}(x, y)$ of X and Y as well as the marginal distributions of X and Y .
2. Use your result from part (1) to compute $P(X \leq 2|Y = 4)$.

Problem 5 (10 points) A health study tracked a group of people for five years. At the beginning of the study, 20% were classified as heavy smokers, 30% as light smokers, and 50% as nonsmokers.

Results of the study showed that light smokers were twice as likely as nonsmokers to die during the five-year study, but only half as likely as heavy smokers.

A randomly selected participant from the study died over the five-year period. Calculate the probability that the participant was a heavy smoker.

Problem 6 (10 points) An insurance company examines its pool of auto insurance customers and gathers the following information:

1. All customers insure at least one car.
2. 70% of the customers insure more than one car.
3. 20% of the customers insure a sports car.
4. Of those customers who insure more than one car 15% insure a sports car.

Calculate the probability that a randomly selected customer insures exactly one car and that car is not a sports car.

Problem 7 (20 points) Let X and Y be independent random variables, both having the geometric distribution with parameter p . Determine and identify the PMF of the random variable

1. $X + Y$
2. $\min\{X, Y\}$

(Extra Credit 10 points) Amy enjoys cooking and eating spaghetti. One day, she dropped a piece of uncooked spaghetti of length 1 and found out that it broke into three smaller pieces. What is the probability that the three pieces happen to form a triangle. (Recall that the lengths of any two sides of a triangle must add to more than the length of the third side.)