Practice Test, Topology, Autumn 2011

Instructions: Answer two of the following three questions. Each question is worth 10 marks. This test will be marked, to provide you with feedback regarding how well you are understanding the material, but it will not counts towards your assessment for the course.

Question 1

(i) Carefully define what it means for a set to be open in the finite complement topology. [**1 mark**]

(ii) Prove that the finite complement topology is, in fact, a topology. [**4 marks**]

(iii) Is $\mathbb{R}$, equipped with the finite complement topology, compact? Provide a proof supporting your answer. [**5 marks**]

Question 2

(i) Give a careful definition of what it means for a set to be closed. [**1 mark**]

(ii) (a) Is the set $A = \mathbb{Q} \cap (1, 2)$, considered as a subset of the real line with the usual topology, closed? Provide a brief argument to support your answer. [**2 marks**]

(b) Consider the space $X = \mathbb{R}$, with the particular point topology for the particular point zero. What is the closure of $(-1, 1)$, considered as a subset of X? Provide a brief argument to support your answer. [**2 marks**]

(iii) Given a continuous function f from the real line to itself (with the usual topology in both cases), prove that the set $E = \{x : f(x) = x\}$ is closed. [**5 marks**]

Question 3

(i) Carefully state the definition of a continuous function. [**1 mark**]

(ii) Consider the function $f : X \to Y$, where X is the real line with the discrete topology and Y is the real line with the usual topology, defined by

$$f(x) = \begin{cases} 1 & \text{if} \quad x \geq 0 \\ 0 & \text{if} \quad x < 0 \end{cases}$$

Is f continuous? Provide a brief argument to support your answer. [**3 marks**]

(iii) (a) Suppose $f : X \to Y$ is one-to-one and continuous and Y is Hausdorff. Prove that X is Hausdorff. [**3 marks**]

(b) Use part a) to prove that the property of a space being Hausdorff is a topological invariant. [**3 marks**]