Tutorial Sheet 1, Topology 2011

1. Let Γ be a graph. Prove that $v(\Gamma) - e(\Gamma) \leq 1$. Furthermore, show that $v(\Gamma) - e(\Gamma) = 1$ if and only if Γ is a tree.

2. Show that any graph contains a tree which includes all the vertices.

3. Let P be a polyhedron satisfying the assumptions of Euler's Theorem. Suppose that each face of P is a regular polygon with p edges and that q faces meet at each vertex. Use Euler's formula to prove that
$$\frac{1}{p} + \frac{1}{q} = \frac{1}{2} + \frac{1}{e}.$$
Use this result to conclude that there are only 5 such polyhedra.

4. Find a continuous function that maps the hyperboloid to the annulus,
$$f : \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 - z^2 = 1\} \to \{(x, y) \in \mathbb{R}^2 : 1 < x^2 + y^2 < 3\}.$$
Suggestion: write the hyperboloid in cylindrical coordinates (r, θ, z) and the annulus in polar coordinates (r, θ). Map the set $\{\theta = 0\}$ in the hyperboloid to the set $\{\theta = 0\}$ in the annulus via
$$g(r, 0, z) = \left(\frac{z}{1 + |z|} + 2, 0\right).$$
Do this for each value of θ, and check that the resulting function is continuous. Does your function have an inverse? If so, is it continuous?