Tutorial Sheet 2, Topology 2011

1. Let X be any set and $p \in X$ be some point in X. Define τ to be the collection of all subsets of X that do not contain p, plus X itself. Prove that τ is a topology on X. (It is called the "excluded point topology.")

2. Consider the following metrics on $\mathbb{R}^2$ (which are not the usual metric): $d_1(x,y) = \max_{i=1,2} |x_i - y_i|$ and $d_2(x,y) = |x_1 - y_1| + |x_2 - y_2|$. Describe the open sets induced by these metrics. (What does an open ball look like?)

3. Let (X,d) be a metric space containing at least two points. Prove that the metric topology cannot be the trivial topology.

4. Prove that, in the real line with the usual topology, every point is a limit point of the rationals.

5. Find all the limit points of the following subsets of the real line:

   (a) $\{(1/m) + (1/n) : n, m = 1, 2, 3, \dots\}$

   (b) $\{(1/n) \sin n : n = 1, 2, 3, \dots\}$

6. Let X be the real line equipped with the finite complement topology. Prove that if A is an infinite set, then every point is a limit point of A. In addition, prove that if A is a finite set, then it has no limit points.

7. Find a family of closed subsets of the real line whose union is not closed.