Tutorial Sheet 9, Topology 2011

1. Prove the following theorem, which was stated in class: If $h : X \to Y$ and $k : Y \to Z$ are continuous functions with $h(x_0) = y_0$ and $k(y_0) = z_0$ for some $x_0 \in X$, then $(k \circ h)_* = k_* \circ h_*$. Also, if $i : X \to X$ is the identity, then i_* is the identity.

2. Describe the homomorphism $f_* : \pi_1(S^1, 1) \to \pi_1(S^1, f(1))$ for each of the following continuous functions:

   (a) $f(e^{i\theta}) = e^{i(\theta+\pi)}$, $\theta \in [0, 2\pi]$

   (b) $f(e^{in\theta}) = e^{i(\theta+\pi)}$, $\theta \in [0, 2\pi]$, $n \in \mathbb{Z}$

   (c)
   $$f(e^{in\theta}) = \begin{cases} e^{i\theta} & \text{if} \quad \theta \in [0, \pi] \\ e^{i(2\pi-\theta)} & \text{if} \quad \theta \in [\pi, 2\pi] \end{cases}$$

3. Compute the fundamental groups of the following spaces:

   (a) A cylinder: $\pi_1(S^1 \times J, x_0)$ where J is any interval in $\mathbb{R}$ and $x_0 = (y_0, z_0) \in S^1 \times J$ is any point.

   (b) The punctured plane: $\pi_1(\mathbb{R}^2 \setminus \{(0, 0)\}, x_0)$, for any point $x_0 \in \mathbb{R}^2 \setminus \{0\}$.

   (c) A punctured disk: $\pi_1(B^2 \setminus \{0\}, x_0)$, for any point $x_0 \in B^2 \setminus \{0\}$.