Practice Test, Topology, Autumn 2013

Instructions: Answer two of the following three questions. Each question is worth 10 marks. This test will be marked, to provide you with feedback regarding how well you are understanding the material, but it will not counts towards your assessment for the course.

Question 1

(i) Carefully state the definition of the finite complement topology. [**2 marks**]

(ii) Prove that the finite complement topology is, in fact, a topology. [**4 marks**]

(iii) Is $\mathbb{R}$, equipped with the finite complement topology, compact? Provide an argument to support your answer. [**4 marks**]

Question 2

(i) Carefully state the definition of a closed set. [**2 marks**]

(ii) Consider the set $A = \mathbb{Q}$, where $\mathbb{Q}$ denotes the rational numbers, as a subset of the real line with the usual topology. Is A closed? Provide an argument to support your answer. [**2 marks**]

(iii) Is the set $A = [0, 1]$, considered as a subset of $\mathbb{R}$ with the trivial topology, closed? Provide an argument to support your answer. [**2 marks**]

(iv) Consider the space $X = \mathbb{R}$, with the particular point topology for the particular point zero. What is the closure of $(-1, 1)$, considered as a subset of X? Provide an argument to support your answer. [**4 marks**]

Question 3

(i) Carefully state the definition of a continuous function. [**2 marks**]

(ii) Consider the function $f : X \to Y$, where X is the real line with the discrete topology and Y is the real line with the usual topology, defined by

$$f(x) = \begin{cases} 1 & \text{if} \quad x \geq 0 \\ 0 & \text{if} \quad x < 0 \end{cases}$$

Is f continuous? Provide an argument to support your answer. [**2 marks**]

(iii) Recall the fact that, if $f : X \to Y$ is continuous and $C \subset X$ is compact, then $f(C)$ is compact. Use this fact to prove that compactness is a topological invariant. [**2 marks**]

(iv) Suppose $f : X \to Y$ is one-to-one and continuous and Y is Hausdorff. Prove that X is Hausdorff. [**4 marks**]