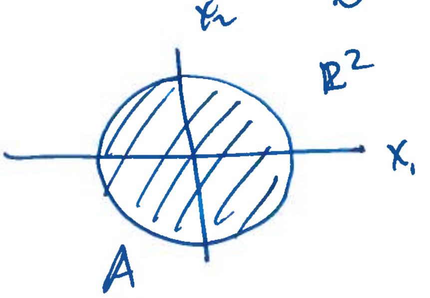
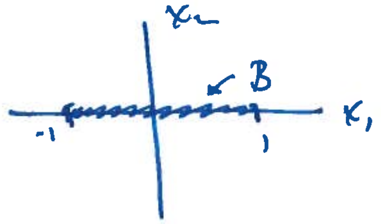
Revision lecture

→ Properties of a space / set depend very much on the topology!

<u>Example:</u> Let $A = \{x \in \mathbb{R}^2 : |x| \leq 1\}$

$B = \{x \in A : (x_1, x_2) = x \text{ s.t. } x_2 = 0\}$





i) Let A have the usual subspace topology inherited from $\mathbb{R}^2$.

a) Is A compact? Yes, Heine-Borel.

b) Is A connected? Yes, eg b/c path connected.

c) Is B open (in A)? No! Can't put a ball in it.

d) Is B closed (in A)? Yes! $A \setminus B$ is open - can always fit a ball in.

ii) Let A have the particular point topology with p.p. $(0,0)$.

a) Is A compact? No. The open cover $\{O_x = \{(0,0), (x_1, x_2)\}\}_{x \in A}$ has no finite subcover.

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b) Is A connected? Yes. No two open sets are disjoint (all contain $(0,0)$), so it's not possible to form a disconnection of A (i.e. U, V open, disjoint, nonempty, ~~[illegible]~~ $U \cup V = A$)

c) Is B open? Yes, $(0,0) \in B$.

d) Is B closed? No. $(0,0) \in A \setminus B$, so its complement is not ~~[illegible]~~ open.

→ Topological invariants:

- compactness, Hausdorff, connectedness, path-connectedness, fund. group.
- <u>not</u>: open/closed, bdd

<u>Ex</u>:

1) Are $A = (1,2) \cup (2,3)$ and $B = \mathbb{R}$ homeomorphic?
(w/ usual sub. top)

- <u>Correct Answer w/ correct reason</u>: No. B is connected + A isn't and connectedness is a top. invariant.
- <u>Incorrect reason</u> ~~Answer~~: A is bdd and B isn't.

2) $X = \mathbb{R}$ w/ small topology
$Y = \mathbb{R}$ w/ finite comp. topology

are they homeomorphic?

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No: $\mathbb{R}^X$ is not compact (by Heine-Borel)
and Y is compact b/c any
set w/ finite comp. top. is compact (def. sheet)