Tutorial Sheet 1, Topology 2013

1. Prove that the finite complement topology, which was defined in lecture, is indeed a topology.

2. Let X be any set and $p \in X$ be some point in X. Define τ to be the collection of all subsets of X that do not contain p, plus X itself. Prove that τ is a topology on X. (It is called the "excluded point topology.")

3. Consider the following metrics on $\mathbb{R}^2$ (which are not the usual metric): $d_1(x, y) = \max_{i=1,2} |x_i - y_i|$ and $d_2(x, y) = |x_1 - y_1| + |x_2 - y_2|$. Draw a picture of what $B_\epsilon(x)$ looks like for each of these metrics.

4. Prove that the metric topology is indeed a topology.

5. Let (X, d) be a metric space containing at least two points. Prove that the metric topology is not the trivial topology. (Recall the trivial topology was defined in lecture.)

6. Prove that the subspace topology is indeed a topology.