Tutorial Sheet 2, Topology 2013

1. Find a countable basis (ie a basis with countably many elements) for the usual topology on $\mathbb{R}$.

2. Prove that, on the real line with the usual topology, every point is a limit point of the rationals.

3. Find all the limit points of the following subsets of the real line (with the usual topology). Explain why such points are limit points and why there are no others (but you don't need to give a formal proof).

   (a) $\{(1/m) + (1/n) : n, m = 1, 2, 3, \dots\}$

   (b) $\{(1/n) \sin n : n = 1, 2, 3, \dots\}$

4. Let X be the real line equipped with the finite complement topology. Prove that if A is an infinite set, then every point is a limit point of A. In addition, prove that if A is a finite set, then it has no limit points.

5. Find a family of closed subsets of the real line whose union is not closed.

6. Verify the following for arbitrary subsets A and B of a topological space X: $\overline{A \cap B} \subseteq \bar{A} \cap \bar{B}$. Show that equality need not hold.

7. Determine the interior, closure, and frontier of each of the following sets.

   (a) The plane with both axes removed.

   (b) $\mathbb{R}^2 \setminus \{(x, \sin(1/x)) : x > 0\}$