Tutorial Sheet 3, Topology 2013

1. Prove that $f : X \to Y$ is continuous if and only if $C \subset Y$ being closed implies $f^{-1}(C) \subset X$ is also closed.

2. Suppose that $f : X \to Y$ is continuous, $A \subset X$, and p is a limit point of A. Prove that $f(p) \in \overline{f(A)}$.

3. Let $f : \mathbb{R} \to \mathbb{R}$ (with the usual topologies) be continuous and define $g : \mathbb{R} \to \mathbb{R}^2$ (with the usual topologies) to be $g(x) = (x, f(x))$. Prove that g is continuous.

4. Let $X = \mathbb{R}$ with the finite complement topology and let $Y = \mathbb{R}$ with the usual topology. Let $f : X \to Y$, $f(x) = x$. Is f continuous? If f^{-1} continuous? Justify your answers.

5. Prove that any two open intervals in the real line (with the usual subspace topology) of the form (a, b) and (c, d) are homeomorphic.

6. Prove that a disc and an ellipse (both with the usual subspace topology) are homeomorphic. Recall that
$$D = \{(x, y) : x^2 + y^2 \leq R^2\}, \qquad E = \left\{(x, y) : \frac{x^2}{a^2} + \frac{y^2}{b^2} \leq 1\right\}$$
for some $R, a, b > 0$.