Tutorial Sheet 4, Topology 2013

1. Find an open cover of $\mathbb{R}$ that does not contain a finite subcover. Do the same for $(0,1)$ considered as a subset of $\mathbb{R}$.

2. Are either of the following sets compact?

   (a) The rational numbers, considered as a subset of the real line.

   (b) $S^n = \{x \in \mathbb{R}^{n+1} : |x| = 1\}$, with finitely many points removed, considered as a subset of $\mathbb{R}^{n+1}$.

3. Let X be an infinite set with the finite complement topology.

   (a) Prove that X is not Hausdorff.

   (b) Prove that every subset of X is compact.

   (c) Find an example of a subset of X that is not closed, which is therefore an example of a compact set that is not closed.

4. Find an example of a function $f : X \to Y$, where X is compact and f is a continuos bijection, but such that f is not a homeomorphism. (Note: based on the theorem from class this implies Y cannot be Hausdorff.)

5. Suppose $f : X \to \mathbb{R}$ where X is compact and f is continuous. Prove that f is bounded and attains its bounds. (This means you should prove that $f(X) \subseteq [a,b]$ for some $a, b \in \mathbb{R}$ and $\exists\ x, y \in X$ such that $f(x) = a$ and $f(y) = b$. Hint: use the Heine-Borel theorem.)